Self Study Series

Important Articles

of the Textbook

PHYSICS XI

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INTRODUCTION

The vital portion of your Text book of Physics for class XI is included in this book of 'Important Articles'.

Here no attempt is made to write extra information or high knowledge to impress students. Articles are written in brief, no details, but to the point, hoping you will not miss the main points in your exam papers.

Foot notes and side notes are not for reproducing in the exams. They are written just for understanding the related article.

Text and figures are made in such a way so that you can reproduce easily in the exams. Thirty one (31) articles have been included for your study.

If you stuck! Just prepare this book, to go through your exams.

Best of luck.

CONTENTS

1- ADDITION OF VECTORS BY RECTANGULAR COMPONENTS

We will devise a formula for addition of more than two vectors, starting from geometrical work to trigonometric.

Consider two vectors,

$$\vec{A}_1 = \vec{OP}$$

$$\vec{A}_2 = \vec{OQ}$$

or $$\vec{A} = \vec{A}_1 + \vec{A}_2$$

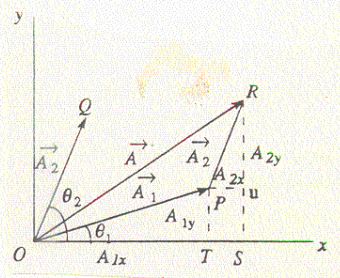


Fig. (1)

We have done some geometrical work, such as, $\vec{A}_2 = \vec{OQ} = \vec{PR}$

$\vec{AU}$ is// $\vec{AT}$ and $\vec{RS}$ are perpendicular to Ox, to Ox, so that PUTS is parallelogram, so

$$\overline{PU} = \overline{TS} = A_{2x}$$

$$\overline{PT} = \overline{US} = A_{1y}$$

and

$$\overline{OT} = A_{1x}$$

$$\overline{RU} = A_{2y}$$

Also $$\overline{OT} + \overline{TS} = \overline{OS}$$

or $$A_{1x} + A_{2x} = A_x \qquad \text{.....} \quad (1)$$

and $$\overline{SU} + \overline{UR} = \overline{SR}$$

or $$A_{1y} + A_{2y} = A_y \qquad \text{.....} \quad (2)$$

Equations (1) & (2) can be written as

$$A_x = A_{1x} + A_{2x} \qquad \text{.....}(3)$$

$$A_y = A_{1y} + A_{2y} \qquad \text{.....}(4)$$

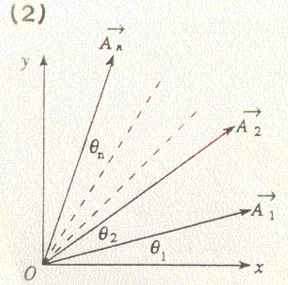


Fig.(2)

From fig.2, taking the case of n vectors, $\vec{A}_1, \vec{A}_2, \vec{A}_3, \ldots \vec{A}_n$ making angles $\theta_1, \theta_2, \theta_3, \ldots \theta_n$ with x-axis respectively.

Generalazing the equation (3), we get

$$A_x = A_{1x} + A_{2x} + A_{3x} + \ldots\ldots + A_{nx}$$

or $$A\cos\theta = A_1\cos\theta_1 + A_2\cos\theta_2 + \ldots\ldots + A_n\cos\theta_n$$

so $\begin{cases} A_x = A\cos\theta \\ A_{1x} = A_1\cos\theta_1 \end{cases}$

or $A\cos\theta = \sum_{i=1}^{n} A_r \cos\theta_r = A_x$ ...... (5)

Similarly generalizing eq.(4), we get

$$A_y = A_{1y} + A_{2y} + A_{3y} + \ldots + A_{ny}$$

or $A\sin\theta = A_1\sin\theta_1 + A_2\sin\theta_2 + \ldots + A_n\sin\theta_n$

[$A_y = A\sin\theta$ so $A_{1y} = A_1\sin\theta_1$

or $A\sin\theta = \sum_{i=1}^{n} A_r\sin\theta_r = A_y$ ....(6)

Squaring and then adding eqs. (5) & (6), we get

$$A^2\cos^2\theta + A^2\sin^2\theta = A_x^2 + A_y^2$$

or $A^2(\sin^2\theta + \cos^2\theta) = A_x^2 + A_y^2$

or $A^2 = A_x^2 + A_y^2$ [$\sin^2\theta + \cos^2\theta = 1$

or $\boxed{A = \sqrt{A_x^2 + A_y^2}}$ ...... (7)

& $\boxed{\tan\theta = \frac{A_y}{A_x}}$ ...... (8)

Therefore to determine the resultant of vectors,

i) Find x- and y-components of vectors,

ii) Add all the x-components to determine resultant A_x,

iii) Add all the y-components to determine resultant A_y,

iv) From eqs. (7) & (8), find the magnitude and direction of the resultant vector.

2- MULTIPLICATION OF TWO VECTORS

We define

Scalar Product (or Dot Product):

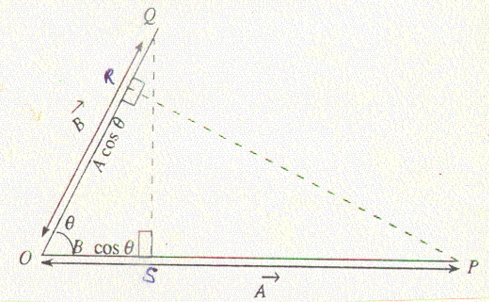
"The scalar or dot product of vectors $\vec{A}$ and $\vec{B}$ is the scalar quantity obtained by multiplying the product of the magnitudes of the vectors by the cosine of the angle between them".

Mathematically, $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$ ...... (1)

The two vectors $\vec{A}$ and $\vec{B}$ are shown in the figure. And

SQ is perpendicular to OP

& RP is perpendicular to OQ



such that

OS = B cos θ = magnitude of the component of $\vec{B}$ along $\vec{A}$....(2)

& OR = A cos θ = " " " " " $\vec{A}$ " $\vec{B}$....(3)

so $\vec{A}.\vec{B}$ = AB cos θ ...... (4)

& $\vec{B}.\vec{A}$ = BA cos θ = AB cos θ ...... (5)

therefore $\vec{A}.\vec{B} = \vec{B}.\vec{A}$ ...... (6)

From eqs. (2) to (6) we conclude that the scalar product of two vectors is the product of the modulus of either vector and the magnitude of the component of the other along the direction of the first vector.

Now we have

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \quad ..... (7)$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \quad ..... (8)$$

then

$$\vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x\hat{i}.\hat{i} + A_xB_y\hat{i}.\hat{j} + A_xB_z\hat{i}.\hat{k} + A_yB_x\hat{j}.\hat{i} + A_yB_y\hat{j}.\hat{j}$$

$$+ A_yB_z\hat{j}.\hat{k} + A_zB_x\hat{k}.\hat{i} + A_zB_y\hat{k}.\hat{j} + A_zB_z\hat{k}.\hat{k}$$

so $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$ ....(9) $\left[\begin{array}{l}\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0 \\ \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1\end{array}\right\}$ — 9(a)

From eq.(9) we see that the scalar product of two vectors is equal to the sum of the products of their corresponding components.

CHARACTERISTICS OF SCALAR PRODUCT

i) From eq.(6) we see that scalar product is commutative,

i.e. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

ii) From $\vec{A}.\vec{B} = 0$, we conclude that

a) either of the two vectors $\vec{A}$ or $\vec{B}$ is null vector

or b) the vectors are mutually perpendicular, such as

$$\vec{A}.\vec{B} = AB \cos 90° = 0$$

iii) For the vectors $\vec{A}$ and $\vec{B}$ are parallel or antiparallel

$$\vec{A}.\vec{B} = AB \cos 0° = AB$$

& $$\vec{A}.\vec{B} = AB \cos 180° = -AB$$

iv) The scalar product obeys associative law, i.e.

$$(m\vec{A}).(n\vec{B}) = mn\vec{A}.\vec{B} = \vec{A}.mn\vec{B} \ ..... \ (10)$$

v) The scalar product is distributive with respect to addition,

i.e. $\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

to prove, we have from eq.

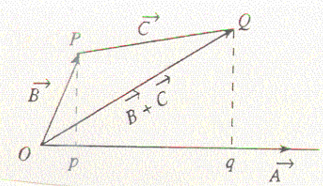
$$\vec{A}.\vec{B} = AB \cos \theta = A(B \cos \theta) ...(11)$$

In the fig., we have

$Op = B \cos \theta$ ....(12) [see eqs.(2) & (3)

$pq = C \cos \theta$ .... (13)

$Oq = (B + C)\cos \theta$ ..... (14)



And

$Oq = op + pq$

multiplying both sides by A,

$$A(Oq) = A(Op) + A(pq) \ .... \ (15)$$

From eqs. (12) to (15) we get

$$A(B + C)\cos \theta = AB \cos \theta + AC \cos \theta \ ....(16)$$

From the definition of dot product, we get

$$\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C} \ .... \ (17)$$

which is the distributive law.

EXAMPLE:

Product of force and displacement which is work. It is a scalar quantity, i.e.,

$$\text{Work} = \vec{F}.\vec{S} = FS \cos \theta \ ....(18)$$

3 - VECTOR PRODUCT

We define

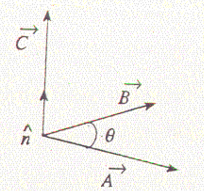
Vector Product (or Cross Product):

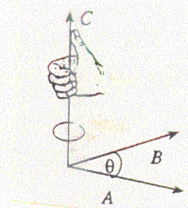
" The vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined to be a vector such that;

i) its magnitude is AB sin θ, θ being the angle between $\vec{A}$ and $\vec{B}$,

ii) its direction is perpendicular to the plane of $\vec{A}$ and $\vec{B}$, and can be determined by right-hand rule.

Mathematically, $\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \, \hat{n}$ ..... (19)





Right-hand rule (in Vector Product):

First place together the tails of the two vectors. Then rotate the vector that occurs first in the product into the second vector through the smaller of the two possible angles. Curl the fingers of the right hand along the direction of rotation. The direction of the thumb will represent direction of the vector product.

Now we have $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ ...... (20)

& $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ ...... (21)

then

$$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x\hat{i}\times\hat{i} + A_xB_y\hat{i}\times\hat{j} + A_xB_z\hat{i}\times\hat{k} + A_yB_x\hat{j}\times\hat{i} + A_yB_y\hat{j}\times\hat{j}$$
$$+ A_yB_z\hat{j}\times\hat{k} + A_zB_x\hat{k}\times\hat{i} + A_zB_y\hat{k}\times\hat{j} + A_zB_z\hat{k}\times\hat{k}$$

since $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$

& $\hat{j}\times\hat{i} = -\hat{k}$, $\hat{k}\times\hat{j} = -\hat{i}$, $\hat{i}\times\hat{k} = -\hat{j}$ ..... (22)

& $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

so $\vec{A}\times\vec{B} = A_xB_y\hat{k} - A_xB_z\hat{j} - A_yB_x\hat{k} + A_yB_z\hat{i} + A_zB_x\hat{j} - A_zB_y\hat{i}$

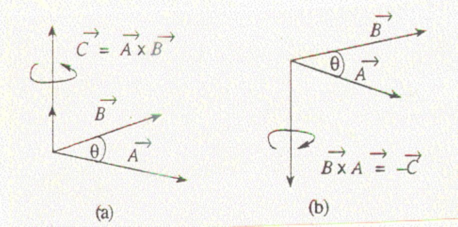
or $\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$..(23)

CHARACTERISTICS OF VECTOR PRODUCT

i) The vector product is non-commutative, i.e.,

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

In the fig. it is illustrated that when the order of the vector product is reversed, the sign of the vector is also reversed.



ii) The vector product is associative, i.e.,

$$(m\vec{A} \times \vec{B}) = \vec{A} \times (m\vec{B}) = m(\vec{A} \times \vec{B})$$

iii) The vector product is distributive with respect to addition, i.e.,

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

iv) For $\vec{A} \times \vec{B} = 0$,

a) either of the two vectors is a null vector,

or b) the two vectors are parallel, such as

$$\vec{A} \times \vec{B} = AB \sin 0^\circ = 0$$

v) For A and B are perpendicular to each other then

$$\vec{A} \times \vec{B} = AB \sin 90^\circ \hat{n} = AB \hat{n}$$ [see eqs.(22)

EXAMPLES:

i) Force on a charged particle moving in a magnetic field, i.e.,

$$\vec{F} = q\vec{v} \times \vec{B}$$

ii) Torque, i.e.,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

4-LAWS OF MOTION

Newton's three laws express mathematical relationship among force, mass and motion of a body.

First Law of Motion:

Statement: "A body continues its state of rest or uniform motion in a straight line unless it is compelled by an unbalanced force impressed upon it".

We define:

Inertia: "The property of a body that opposes any change in its state of motion or rest".

Newton's first law introduces the idea of force as an agent causing a body to change its state of motion or rest.

EXAMPLES:

1. A standing car will remain standing unless some force is applied.
2. A fast moving bus, on application of brakes, comes to a halt but the passengers and other loose objects will tend to continue their motion thats why they will be thrown forward.
3. A bomb dropped from an aeroplane does not fall vertically but describes a curved path.
4. When a space ship is launched at altitude, it tends to move with constant speed.

Second Law of Motion:

Statement: "The effect of an applied force on a body is to cause it to accelerate in the direction of the force. The acceleration is in direct proportion to the force and is inversely proportional to the mass of the body".

Mathematically,

$$\vec{F} = m\vec{a} \quad ..... \quad (1)$$

It tells us that when a force is applied to a body, it moves in the direction of force and will move more faster as the applied force remain in function.

In eq.(1) m is proportionality constant and is the inertial mass of the object. Also this equation tells us for a fixed force, the larger the mass of a body, the smaller its acceleration.

EXAMPLES:

1. When a force is applied on a body at rest, it moves in the direction of force.
2. For two balls--rubber and lead--- the same force of kick will produce more acceleration in the rubber ball.
3. When a paratrooper jumps out of an aeroplane, before the opening of his parachute, he gains acceleration due to his weight.

Third Law of Motion:

Statement: "To every action (force) there is always an equal and opposite reaction (force)".

Newton's third law tells us that a body cannot experience a force from its environments without exerting an equal and opposite force on its environments. Forces in nature always occur in pairs.

EXAMPLES:

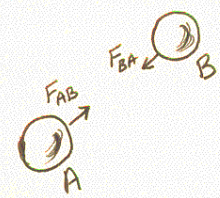
1. In the fig., there is interaction between bodies A and B.

   $\vec{F}_{AB}$ = force exerted by A on B

   $\vec{F}_{BA}$ = force exerted by B on A

   then

   $$\vec{F}_{AB} = -\vec{F}_{BA}$$



2. In jumping off the ground, we exert a force on the ground then an oppositely directed force by the ground is exerted on us.

3. A paratrooper descending with uniform velocity. Here the force of gravity is balanced by the reaction of air on the parachute.

4. A person holding a body with a string. The tension and weight are balanced.

5- ELASTIC COLLISIONS IN ONE DIMENSION

Let

m_1 = mass of first body

m_2 = mass of second body

v_1 = velocity before collision of first body

v_2 = velocity before collision of second body

v_1' = velocity after collision of first body

v_2' = velocity after collision of second body

We have from the law of conservation of momentum,

total momentum before collision = total momentum after collision

$$\text{or } m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \qquad \ldots\ldots \quad (1)$$

$$\text{or } m_1(v_1 - v_1') = m_2(v_2' - v_2) \qquad \ldots\ldots \quad (2)$$

Also from the law of conservation of energy,

total K.E. before collision = total K.E. after collision

$$\text{or } \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2 \qquad \ldots\ldots \quad (3)$$

$$\text{or } m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

$$\text{or } m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2) \qquad \ldots\ldots \quad (4)$$

Dividing eq. (4) by eq. (2), we get

$$v_1 + v_1' = v_2' + v_2$$

$$\text{or } v_1 - v_2 = -(v_1' - v_2') \qquad \ldots\ldots \quad (5)$$

or we conclude

Speed of approach = Speed of recession

<u>Special cases:</u>

<u>Case a)</u>: When $m_1 = m_2$

from eq. (1), we get

$$v_1 + v_2 = v_1' + v_2' \qquad \ldots\ldots \quad (6)$$

Adding eqs. (5) & (6), we get

$$2\ v_1 \ = \ 2\ v_2'$$

$$\text{or } v_1 = v_2'$$

$$\text{so } v_2' \ = \ v_1$$

putting this value of v_1 in eq. (6), we get

$$\cancel{v_2'} + v_2 = v_1 + \cancel{v_2'}$$

$$\text{or } v_2 \ = \ v_1'$$

$$\text{so } v_1' \ = \ v_2$$

We conclude

When two particles of equal mass collide elastically, they exchange their velocities.

<u>Case b</u>): When $v_2 = 0$

From eqs. (1) and (5), we get

$$m_1 v_1 \ = \ m_1 v_1' \ + \ m_2 v_2' \qquad \ldots\ldots \ (7)$$

$$\& \qquad v_1 \ = \ v_2' \ - \ v_1' \qquad \ldots\ldots \ (8)$$

multiplying eq. (8) with m_1, we get

$$m_1 v_1 \ = \ m_1 v_2' \ - \ m_1 v_1' \qquad \ldots\ldots \ (9)$$

Adding eqs. (7) & (9), we get

$$2\ m_1 v_1 \ = \ m_2 v_2' \ + \ m_1 v_2'$$

$$\text{or} \quad 2\ m_1 v_1 \ = \ v_2' \,(\,m_1 + m_2)$$

$$\text{or} \quad v_2' \ = \ \frac{2\ m_1}{m_1 + m_2}\ v_1 \qquad \ldots\ldots \ (10)$$

putting this value of v_2 in eq. (8), we get

$$v_1 \ = \ \frac{2\ m_1}{m_1 + m_2}\ v_1 \ - \ v_1'$$

$$\text{or} \quad v_1' \ = \ \frac{2\ m_1}{m_1 + m_2}\ v_1 \ - \ v_1$$

$$\text{or} \quad v_1' \ = \ \frac{2m_1 - m_1 - m_2}{m_1 + m_2}\ v_1$$

$$\text{or} \quad v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \qquad \ldots\ldots\ldots (11)$$

already we have

$$v_2' = \frac{2\, m_1}{m_1 + m_2} v_1 \qquad \ldots\ldots\ldots (10)$$

A) When $m_1 = m_2$

From eqs, (10) & (11), we get

$$\left.\begin{aligned} & v_1' = 0 \\ \& \quad & v_2' = v_1 \end{aligned}\right. \qquad \ldots\ldots\ldots (12)$$

We conclude:

The incident particle which was moving with v_1, comes to rest while the target particle that was at rest begins to move with velocity v_1.

B) When $m_2 >> m_1$

From eqs. (10) & (11), we get

$$\left.\begin{aligned} & v_1 \simeq - v_1 \\ \& \quad & v_2' \simeq 0 \end{aligned}\right\} \qquad \ldots\ldots\ldots (13)$$

We conclude:

The small incident particle just bounces off in the opposite direction while the heavy target remains almost motionless.

C) When $m_2 << m_1$

From eqs. (10) & (11), we get

$$\begin{aligned} & v_1' \simeq v_1 \\ & v_2' \simeq 2\, v_1 \end{aligned} \qquad \ldots\ldots\ldots (14)$$

We conclude:

The incident particle keeps on moving without loosing much energy, while the target particle moves with the double velocity.

5-b: To calculate <u>final velocities</u> in terms of initial velocities & masses of the bodies colliding elastically

When two objects collide and the initial and final velocities of both are parallel or anti-parallel the collision is said to be one-dimensional. The collision of two boxcars on a railway track is an example of collisions in one dimension. Generally, the collision of any two bodies that approach head-on and recoil along their original line of motion is one dimensional collision. Although these collisions are exceptional, but they display a simple way of some important features of more complicated collisions.

In an **<u>elastic collision</u>** of two particles moving along a straight line, the laws of conservation of momentum and energy completely determine the final velocities in terms of the initial velocities.
If the net external force on the system of masses is zero so that momentum is also conserved, for one-dimensional collision, then from laws of conservation of energy and momentum, we have two equations.

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \qquad \ldots\ldots (1)$$

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_1 v'^2_1 + \tfrac{1}{2} m_2 v'^2_2 \qquad \ldots\ldots (2)$$

from eqs. (1) & (2), we have

$$m_1 (v_1 - v'_1) = m_2 (v'_2 - v_2) \qquad \ldots\ldots (3)$$

$$m_1 (v_1^2 - v'^2_1) = m_2 (v'^2_2 - v_2^2) \qquad \ldots\ldots (4)$$

dividing eq. (4) with eq. (3), we get

$$v_1 + v'_1 = v'_2 + v_2 \qquad \ldots\ldots (5)$$

multiplying eq. (5) with m_1 and then with m_2, so we get eqs. (6) & (7)

$$m_1(v_1 + v'_1) = m_1(v'_2 + v_2) \qquad \ldots\ldots (6)$$

$$m_2(v_1 + v'_1) = m_2(v'_2 + v_2) \qquad \ldots\ldots (7)$$

Subtracting eq. (3) with eq. (7), we get

$$v'_1 = \frac{(m_1 - m_2)}{m_1 + m_2} v_1 + \frac{2 (m_2)}{m_1 + m_2} v_2 \qquad \ldots\ldots (8)$$

Adding eqs. (3) and (6), we get

$$v'_2 = \frac{2 (m_1)}{m_1 + m_2} v_1 + \frac{(m_2 - m_1)}{m_1 + m_2} v_2 \qquad \ldots\ldots (9)$$

6- MOTION OF A BODY ON AN INCLINED PLANE

We define:

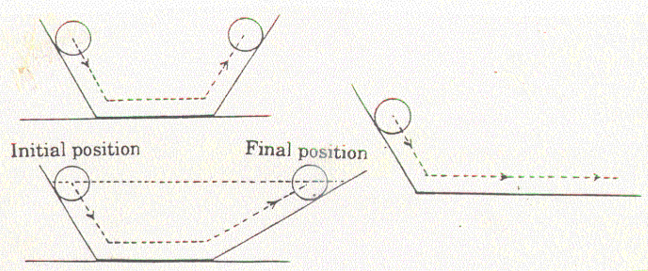
Inclined plane:

"Sloping surface used to reduce the effort of moving a load".

To prove :

Constant motion requires no force. Or motion along a horizontal plane is constant.

Galileo observed the experiment as shown in the fig.



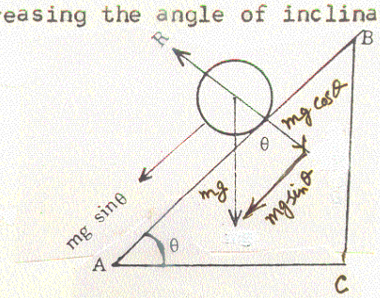
In the case of a plane that slope downward, there is a cause of acceleration. The plane sloping upward there is retardation. In the fig., a ball tends to rise to its original height regardless of the slope.

In case of horizontal plane there should be neither retardation nor acceleration and the motion should be constant.

For the planes with downward and upward slopes, the cause of acceleration and retardation is the force of gravity. However in horizontal plane for constant motion requires no force.

To show that :

The force can be diluted by decreasing the angle of inclination.



Consider a body of mass m sliding along an inclined plane having an angle of inclination θ. Neglecting the force of friction, the force along the plane is,

$$F = mg \sin \theta \quad .....(1)$$

The maximum value of $\sin \theta$ is 1 corresponding to $\theta = 90°$ & the minimum value of $\sin \theta$ is 0 corresponding to $\theta = 0°$.

So $F_{max} = mg \sin 90° = mg$ & $F_{min} = mg \sin 0° = 0$

As θ varied from 90° to 0°, the force varies from a maximum value to zero. i.e.,

the force can be diluted from its maximum value mg to any desired value by selecting the angle of inclination θ.

7- PROJECTILE

We define:

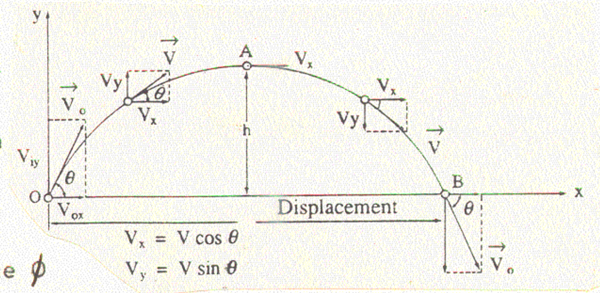
Projectile:

"An object launched in an arbitrary direction in the gravitational field of the earth with the initial velocity having no mechanism of propulsion".

Let

v_i = initial velocity of the projectile

θ = angle of projection with horizontal



To calculate,

the velocity v & angle ϕ (at any time t), we have

$$v_f = v_i + at \quad \& \quad S = v_i t + \tfrac{1}{2}at^2 \quad \ldots\ldots (1)$$

for x-component

$$v_x = v_{ix} \quad \ldots.(2) \quad \& \quad x = v_{ix}t \quad \ldots\ldots (3)$$

& for y-component

$$v_y = v_{iy} - gt \quad \ldots.(4) \quad \& \quad y = v_{iy}t - \tfrac{1}{2}gt^2 \ldots. (5)$$

We have

$$v_x = v_i \cos\theta \quad \ldots\ldots (6) \quad \left[A_x = A\cos\theta \right.$$

$$v_y = v_i \sin\theta - gt \quad \ldots\ldots.. (7)$$

So

$$v = v_x + v_y$$

$$\text{or} \quad v = v_i\cos\theta + (v_i \sin\theta - gt)$$

$$\text{or} \quad v = \sqrt{(v_i\cos\theta)^2 + (v_i\sin\theta - gt)^2} \quad \left[A = A_x^2 + A_y^2 \right.$$

$$= \sqrt{v_i^2\cos^2\theta + v_i^2\sin^2\theta + g^2t^2 - 2v_i gt\sin\theta}$$

$$= \sqrt{v_i^2(\cos^2\theta + \sin^2\theta) - 2gtv_i\sin\theta + g^2t^2}$$

$$\text{or} \quad \boxed{v = \sqrt{v_i^2 - 2gtv_i\sin\theta + g^2t^2}} \quad \ldots\ldots (8)$$

for angle ϕ,

$$\tan\phi = \frac{v_y}{v_x} = \frac{v_i\sin\theta - gt}{v_i\cos\theta}$$

$$\text{or} \quad \boxed{\phi = \tan^{-1}\frac{v_i\sin\theta - gt}{v_i\cos\theta}} \quad \ldots..(9)$$

To calculate the path (trajectory) of the projectile:

We have from eqs. (3) & (5),

$$x = v_i \cos\theta \, t \quad \ldots\ldots (10) \qquad [A_x = A \cos\theta$$

$$\& \quad y = v_i \sin\theta \, t - \tfrac{1}{2} g t^2 \ldots (11) \qquad [A_y = A \sin\theta$$

from eq.(10)

$$t = \frac{x}{v_i \cos\theta}$$

putting this value in eq. (11), we get

$$y = \frac{v_i \sin\theta}{v_i \cos\theta}\, x - \tfrac{1}{2} g \left(\frac{x}{v_i \cos\theta}\right)^2$$

$$\text{or } y = x \tan\theta - \tfrac{1}{2} \frac{g x^2}{v_i^2} \sec^2\theta \quad \ldots\ldots\ldots (12)$$

$$\text{put } a = \tan\theta \;\&\; b = \frac{g}{2 v_i^2} \sec^2\theta$$

a & b being constants, we get

$$y = ax - b x^2 \quad \ldots\ldots\ldots (13)$$

which is the equation of a parabola. So the trajectory of projectile is a parabola.

<u>To calculate maximum height H</u> :

We have from eq. (4)

$$v_y = v_i \sin\theta - gt$$

since v_y becomes zero, so

$$0 = v_i \sin\theta - gt$$

$$\text{or } t = \frac{v_i \sin\theta}{g} \quad \ldots\ldots\ldots (14)$$

We have from eq. (11), as

$$y = v_i \sin\theta \, t - \tfrac{1}{2} g t^2$$

putting $y_{max} = H$ and value of t, we get

$$H = (v_i \sin\theta) \frac{v_i \sin\theta}{g} - \frac{\tfrac{1}{2} g (v_i \sin\theta)^2}{g^2}$$

$$\text{or} \quad H = \frac{v_i^2 \sin^2\theta}{g} - \frac{1}{2} g \frac{v_i^2 \sin^2\theta}{g^2}$$

$$\text{or} \quad \boxed{H = \frac{v_i^2 \sin^2\theta}{2g}} \quad \ldots\ldots \quad (15)$$

<u>To calculate range R of the projectile</u>:

Taking eq. (12)

$$y = x \tan\theta - \frac{1}{2} \frac{g x^2}{v_i^2} \sec^2\theta$$

for horizontal case, y = 0, so

$$0 = x \tan\theta - \frac{1}{2} \frac{g x^2}{v_i^2} \sec^2\theta$$

$$\text{or} \quad \left(\tan\theta - \frac{1}{2} \frac{g x}{v_i^2} \sec^2\theta\right) x = 0$$

Either $x = 0$ [which is point of projection (0,0)

$$\text{or} \quad \tan\theta - \frac{1}{2} \frac{g x}{v_i^2} \sec^2\theta = 0 \quad \ldots\ldots\ldots \quad (16)$$

the value of x = R , the range, so

$$\tan\theta - \frac{1}{2} \frac{g R}{v_i^2} \sec^2\theta = 0$$

$$\text{or} \quad R = \frac{2v_i^2 \tan\theta}{g \sec^2\theta} = \frac{2v_i^2 \sin\theta \times \cos^2\theta}{g \cos\theta}$$

$$\text{or} \quad R = \frac{2\, v_i^2 \sin\theta \cos\theta}{g} \quad \ldots\ldots \quad (17)$$

since $\sin 2\theta = 2 \sin\theta \cos\theta$

$$\text{so} \quad R = \boxed{\frac{v_i^2 \sin 2\theta}{g}} \quad \ldots\ldots\ldots \quad (18)$$

<u>Maximum range, R_{max}</u>:

In eq.(18), v_i and g are constants. For R maximum sin 2 θ should be maximum. And the maximum value of $\sin 2\theta = 1 \Rightarrow 2\theta = 90°$ or $\theta = 45°$

$$\text{So} \quad \boxed{R_{max} = \frac{v_i^2}{g}} \quad \ldots(19)$$

<u>To calculate the time of flight t :</u>

We have $S = vt$

$$\text{or } t = \frac{S}{v}$$

for horizontal case

$$t = \frac{R}{v_x}$$

putting the values from eq. (6) & eq. (17), we get

$$t = \frac{2\, v_i^2 \sin\theta \cos\theta}{g\, v_i \cos\theta}$$

$$\text{or } \boxed{t = \frac{2v_i \sin\theta}{g}} \qquad \ldots\ldots (20)$$

<u>Time to reach at maximum height:</u>

We have from eq.(14) as

$$\boxed{t' = \frac{v_i \sin\theta}{g}} \qquad \ldots\ldots (21)$$

8- CENTRIPETAL ACCELERATION AND CENTRIPETAL FORCE

We define:

Centripetal acceleration: "Acceleration directed towards the centre of a circle."

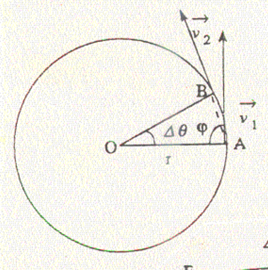
Centripetal force: "A force that causes a body to move in a circular path."

When a body moves along a circular path, its direction of velocity continuously change. From the definition of acceleration change of velocity produces acceleration, which is called centripetal acceleration.

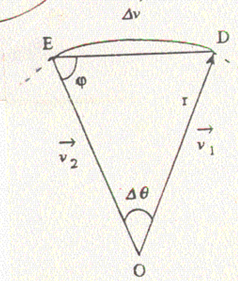
To calculate the magnitude of centripetal acceleration:

Let

m = mass of the stone
 = angular speed of the stone
r = radius of the circle
v = its linear velocity along the tangent
v_1 = velocity at point A
v_2 = velocity at point B



(a)



(b)

since two velocities at points A and B are same, so

$$v_1 = v_2 = v \quad \ldots\ldots(1)$$

from fig(b), we have

$$\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$$

or $$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 \quad \ldots\ldots(2)$$

Now $\angle AOB = \angle DOE = \Delta\theta$

For small angle

chord Δv = arc DE ....(3)

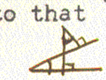
and $\sin\theta = \theta$ ....(4)

We have

$$\sin\theta = \frac{\Delta v}{v_2}$$

A theorem:

Angle between the perpendiculars of the sides of an angle is equal to that angle.



Taylor's series expansion for;

$$\sin\theta = \theta - \frac{\theta^3}{3.2.1} + \frac{\theta^5}{5!} - \ldots$$

putting the values from eqs. (1), (3) & (4), we get

$$\theta = \frac{\Delta v}{v}$$

$$v\,\Delta\theta = \Delta v \quad .....(5)$$

[for small change $\theta = \Delta\theta$

multiplying and dividing by Δt to L.H.S., we get

$$v\,\frac{\Delta\theta}{\Delta t}\,\Delta t = \Delta v$$

$$\text{or} \quad \Delta v = v\,\omega\,t \quad .....(6)$$

[$\omega = \frac{\theta}{t}$

$$\text{or} \quad \frac{\Delta v}{\Delta t} = \omega v \quad .....(7)$$

Now we define

$$\vec{a} = \frac{\Delta \vec{v}}{t} \quad .....(8)$$

so

$$a = \omega v \quad .....(9)$$

$$\text{or} \quad \boxed{a = \omega^2 r = \frac{v^2}{r}} \quad .....(10)$$

[$v = r$ or $= \frac{v}{r}$

In vectorial form

$$\vec{a} = -\omega^2 r = -\frac{v^2}{r^2}\vec{r} \quad .....(11)$$

where negative sign indicates that the acceleration is towards the centre. (Indicated by angle ϕ in fig.(b).

<u>To calculate centripetal force:</u>

We have

$$\vec{F} = m\,\vec{a} \quad ...... (12)$$

from eqs. (11) & (12), we get

$$\vec{F} = -m\,\omega^2\,\vec{r} = -\frac{mv^2}{r^2}\,\vec{r} \quad ......(13)$$

$$\text{or} \quad \boxed{F_c = \frac{mv^2}{r}} \quad ......(14)$$

<u>EXAMPLES</u>:

1. A stone is whirled in a horizontal circle by means of a string.
2. Planets move around the sun.
3. When a racing car moves round a circular track the friction at the wheels provides the centripetal force.

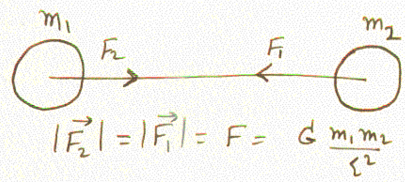
9- NEWTON'S LAW OF GRAVITATION

Statement:

"Everybody in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres".

Explanation:

If two bodies of masses m_1 and m_2 are placed at a distance r from their centres, then the force of attraction is given by



$$F \propto m_1 m_2$$

$$\& \quad F \propto \frac{1}{r^2}$$

Combining the two proportionalities,

$$F \propto \frac{m_1 m_2}{r^2} \qquad \ldots\ldots\ldots \quad (1)$$

$$\text{or} \quad F = G \frac{m_1 m_2}{r^2} \qquad \ldots\ldots\ldots \quad (2)$$

or in vector form, $\vec{F} = - G \frac{m_1 m_2}{r^2} \hat{r}$ .... (3)

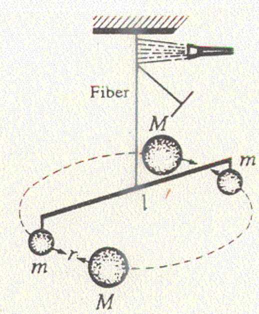
where G is proportionality constant, called constant of gravitation or universal gravitational constant. Negative sign indicates attractive force.

Calculation of G :

In the fig., we have

A light rod having length l is suspended with a quartz fibre. Two identical balls each of mass m suspended from rods' ends. Heavy balls each of mass M are brought near the small balls.

Points A, B & C shows lamp and scale arrangement.



Magnitude of each force between masses M and m is

$$F = \frac{G M m}{r^2} \qquad \ldots\ldots (4)$$

the value of torques are

$$\tau_1 = \frac{G M m}{r^2} \frac{l}{2} \qquad [\tau = r F_\perp$$

$$\tau_2 = \frac{G M m}{r^2} \frac{l}{2}$$

where l/2 is moment arm. (l being length of rod).

So total torque, τ, is

$$\tau = \tau_1 + \tau_1 = \frac{G M m}{r^2} \frac{l}{2} + \frac{G M m}{r^2} \frac{l}{2}$$

$$\text{or} \quad \tau = \frac{G M m}{r^2} l \qquad \ldots\ldots\ldots(5)$$

Now force due to M and m produce torque which causes a twist in the fibre.

The twist θ is proportional to the torque ,

$$\tau \propto \theta$$

$$\text{or} \quad \tau = c\,\theta \qquad \ldots\ldots (6)$$

where c is torsion constant and can be calculated.

From eqs. (5) & (6), we get

$$\frac{G M m}{r^2} l = c\,\theta$$

$$\text{or} \quad G = \frac{c\,\theta r^2}{M m l} \qquad \ldots\ldots (7)$$

θ is measured by lamp and scale arrangement.

c can be calculated from the material of the fibre.

r is distance between M & m.

l is length of the rod.

M & m are masses of heavy and small balls.

The value of G found from this experiment is

$$G = 6.673 \times 10^{-11} \ N\,m^2 kg^{-2}$$

10- WORK DONE AGAINST THE GRAVITATIONAL FORCE

To prove that:

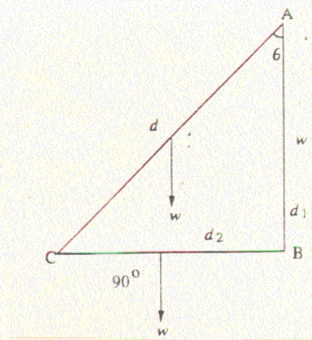
The work done in a gravitational field is independent of the path followed by the body.

or

The total work done in moving a body along a closed path in a gravitational field is always equal to zero.

Let

We have a gravitational field in which a body moves from point A to C through two paths.



To calculate the work done in taking the body from A to C,

i) $W_{A\to C}$, W.done directly from A to C

ii) $W_{A\to B\to C}$, W. done from A to B, then from B to C.

We have

$$W_{A\to C} = \vec{F}.\vec{d} \qquad \left[\vec{F} = \vec{w}\right.$$

$$= w.d$$

$$= wd\cos\theta \qquad \left[\cos\theta = \frac{d_1}{d}\right.$$

so $$W_{A\to C} = w\,d_1 \ldots.(1) \qquad \text{or } d_1 = d\cos\theta$$

And

$$W_{A\to B} = \vec{F}.\vec{d_1}$$

$$= \vec{w}.\vec{d_1} \qquad \left[\theta = 0\right.$$

$$= wd_1\cos\theta \qquad \text{so } \cos\theta = 1$$

or $$W_{A\to B} = w\,d_1 \quad \ldots\ldots. (2)$$

And

$$W_{B\to C} = \vec{F}.\vec{d_2}$$

$$= \vec{w}.\vec{d_2} = w\,d_2\cos\theta \qquad \left[\theta = 90^\circ\right. \quad \cos 90^\circ = 0$$

or $$W_{B\to C} = 0 \quad \ldots\ldots.. (3)$$

So from eqs. (2) & (3), we get

$$W_{A\to B\to C} = w\,d_1 + 0 = w\,d_1 \quad \ldots. (4)$$

From eqs. (1) & (4), we conclude that, the work done in a gravitational field is independent of the path followed by the body.

Now work done in moving a body from C to A is

$$W_{C\to A} = \vec{F}.\vec{d}$$
$$= \vec{w}.\vec{d}$$
$$= w\ d \cos\theta$$

so $W_{C\to A} = -wd_1$ ....(5)

$\left[\begin{array}{l}\theta = 180^\circ \\ \text{or } \cos\theta = -1 \\ \& \ d\cos\theta = d_1\end{array}\right.$

The total work done in moving the body around the closed path ABCA is

$$W_{total} = W_{A\to B} + W_{B\to C} + W_{C\to A}$$

From eqs. (2), (3) & (5), we get

$$W_{total} = w\ d_1 + 0 + (-w\ d_1)$$
$$= w\ d_1 - w\ d_1 = 0$$

so $W_{total} = 0$

Thus, the total work done in moving a body along a closed path in a gravitational field is always equal to zero.

<u>Conservative field</u>:

In which the work done between two points in the field is independent of the path followed between the two points.

<u>Examples of Conservative fields are:</u>

i) Gravitational field
ii) Electric field
iii) Magnetic field.

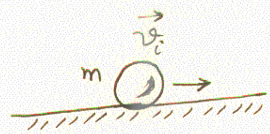
11- KINETIC ENERGY

We define:

"The kinetic energy (K.E.) of a body is the energy possessed by a body due to its motion".

Let

A body of mass m is moving with initial velocity v_i. A constant force F be applied to stop the body, the acceleration produced is



$$a = -\frac{F}{m} \quad [F = ma$$

The body comes to rest after covering a distance d.

So we have

$$v_f = 0$$

$$a = -\frac{F}{m}$$

$$S = d$$

$$v_i = v$$

Using the equation,

$$v_f^2 - v_i^2 = 2 a S$$

putting the values

$$0 - v^2 = 2 \times -\frac{F}{m} \times d$$

$$\text{or} \quad F d = \frac{m v^2}{2}$$

F d is the work done by the body before comming to rest, which must be equal to the energy possessed by the body due to its motion, so

$$\boxed{K.E. = \tfrac{1}{2} m v^2} \quad \ldots\ldots (1)$$

Its a scalar quantity. And the units of kinetic energy are the same as those of work.

12- POTENTIAL ENERGY

We define:

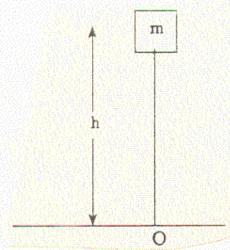
"Potential energy of a body is the energy possessed by it due to its position in a field of force or by its constrained state".

To calculate

Gravitational Potential energy (P.E.),

Let

In the gravitational field, a body having mass m is raised through height h from the ground.



Here the weight is balanced by the force,

$$F = w$$

So the work done against the force of gravity is

$$w\,h = m\,g\,h$$

This work done is due to the change in position, which is called gravitational potential energy,

$$\boxed{P.\,E. = m\,g\,h} \qquad \ldots\ldots (1)$$

Absolute Gravitational Potential Energy :

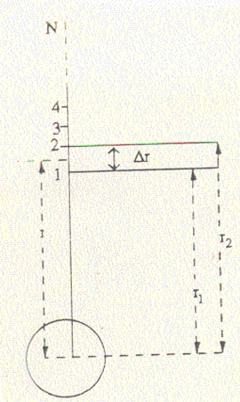
We define:

"Energy required to move a mass from the earth up to an infinite distance".

To calculate the value of absolute gravitational potential energy,

Consider

A body of mass m which moves from point 1 to far off point N with constant velocity in the gravitational field.



As gravitational force changes with distance, so divide the distance between 1 to N into small steps, each of length Δr.

We have

$$r_2 - r_1 = \Delta r \quad \ldots . (2)$$

and the mean distance

$$r = \frac{r_1 + r_2}{2} \quad \ldots ..(3)$$

From eq.(2), we have

$$r_2 = \Delta r + r_1 \qquad \ldots\ldots\ldots (4)$$

putting the value of r_2 from eq.(4) in eq.(3), we get

$$r = \frac{r_1 + \Delta r + r_1}{2}$$

$$\text{or} \quad r^2 = \left(\frac{r_1 + \Delta r + r_1}{2}\right)^2$$

$$= \left(\frac{2 r_1}{2} + \frac{\Delta r}{2}\right)^2$$

$$\text{or } r^2 = r_1^2 + \frac{(\Delta r)^2}{4} + r_1 \Delta r$$

Neglecting $\frac{(\Delta r)^2}{4}$ as $(\Delta r)^2 << r_1^2$,

$$r^2 = r_1^2 + r_1(r_2 - r_1) \qquad \left[\text{from eq. (2)}\right.$$

$$= r_1^2 + r_1 r_2 - r_1^2$$

$$\text{or} \quad r^2 = r_1 r_2 \qquad \ldots\ldots (5)$$

Now, if M_e is the mass of earth, the gravitational force at the centre of the small step is

$$F = G \frac{m M_e}{r^2} \qquad \ldots\ldots (6)$$

From eqs. (5) & (6), we get

$$F = G \frac{m M_e}{r_1 r_2} \qquad \ldots\ldots (7)$$

As this force is assumed to be constant during Δr, so

$$W_{1\rightarrow 2} = G \frac{m M_e}{r_1 r_2} (\Delta r)$$

$$= G \frac{m M_e}{r_1 r_2} (r_2 - r_1)$$

$$\text{or} \quad W_{1\rightarrow 2} = G M_e m \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\text{Similarly} \quad W_{2\rightarrow 3} = G M_e m \left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$W_{3\rightarrow 4} = G M_e m \left(\frac{1}{r_3} - \frac{1}{r_4}\right)$$

::::::::::::::::::::::::::::

$$W_{(N-1)\rightarrow N} = G M_e m \left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

[from eq.(2)
$\Delta r = r_2 - r_1$
So $W_{1\rightarrow 2} = G m M_e \frac{r_2 - r_1}{r_1 r_2}$
$= G m M_e \left(\frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2}\right)$
$= G m M_e \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$]

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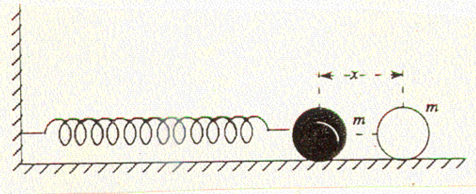
13- MOTION UNDER AN ELASTIC RESTORING FORCE

Consider

A body having mass m attached to one end of a spring and placed on a frictionless horizontal surface.



When the mass is pulled through x,
From modified form of Hook's law, the applied force F is

$$F = k\,x \quad .....(1)$$

where k is spring constant.

Due to elasticity, the spring opposes the applied force. This opposing force is called restoring force,

Restoring force = F = $-kx$...(2)

In fig.2, mass m is pulled towards right with some force, the extension gives rise to restoring force.

Some work will be done in displacing from equilibrium against this force. It will be stored as its potential energy. When released this PE changes to KE. At equilibrium all PE converts to KE. Due to inertia it will move towards left.

When compressed whole KE changes to PE. The process is repeated and the mass continues to oscillate between the extreme positions.

We define, Hook's law as
"Within the limits of perfect elasticity strain is directly proportional to stress".

or stress $\propto$ strain

or $\dfrac{\text{stress}}{\text{strain}}$ = const.

$$\frac{F/a}{l/L} = E$$

$$\frac{FL}{la} = E$$

L, a & E are const., so

$F \propto l$

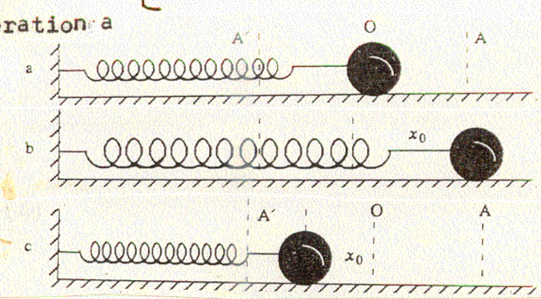
or $F = k\,l$

or $F = k\,x$

We may call it the modified form of Hook's law.

To calculate the acceleration a of the mass, we have

$$F = ma \quad ...... \quad (3)$$



From eqs. (2) & (3), we get

$$-k\,x = m\,a$$

or $\boxed{a = -\dfrac{k}{m}\,x}$ ....(4)

or $a = -(\text{const})\,x$

or $a \propto -x$

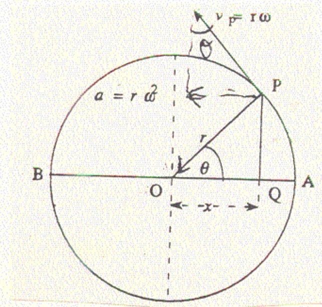
or $a \propto -$ displacement

Such a motion in which acceleration is proportional to the displacement and is directed towards the centre is called <u>Simple harmonic motion</u>.

14- SIMPLE HARMONIC MOTION AND CIRCULAR MOTION

To relate SHM with circular motion

Consider



A point P moving along circular trajectory around the centre O, with angular speed ω.

The radius of the circle is r, speed of the moving point P is

$$v_p = r\omega \quad \ldots\ldots\ldots (1)$$

Consider the motion of the point Q, the projection of P on the diameter AB.

As P describes a constant angular speed ω, Q oscillates to and fro along the diameter. As Q moves away from O, it slows down, & as it moves towards O it speeds up, i.e., its acceleration is directed towards O.

The magnitude of acceleration of the point P is

$$a_c = \frac{v_p}{r} = r\omega^2 \qquad [v_p = r\omega$$

Its component along AOB is

$$a = r\omega^2 \cos\theta$$

Since it is directed towards centre and $x = r\cos\theta$, so

$$\boxed{a = -\omega^2 x} \quad \ldots\ldots (2)$$

Thus the point Q has an acceleration proportional to displacement and directed towards the centre, which is the characteristic of SHM. So the projection of P executes SHM. We can define SHM as " the projection of uniform circular motion upon any diameter of a circle".

To calculate time period T of Q , we have

$$\omega = \frac{\theta}{t} \quad \text{or} \quad t = \frac{\theta}{\omega}, \quad \theta = 2\pi \text{ rad.}$$

$$\text{So} \quad T = \frac{2\pi}{\omega} \quad \ldots\ldots\ldots (3)$$

From the fig., the instantaneous displacement x is

$$\boxed{x = r\cos\theta} \quad \text{or} \quad x = r\cos\omega t \quad \ldots\ldots\ldots (4) \qquad [\theta = \omega t$$

The speed v of the point Q, (from the fig.)

$$v = v_p \sin\theta \quad \text{or} \quad v = r\omega \sin\omega t \quad \ldots\ldots\ldots (5)$$

$$\text{or} \quad v = r\omega\sqrt{1 - \cos^2\omega t} \qquad \left[\begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ \text{or } \sin\theta = 1 - \cos^2\theta \end{array}\right.$$

from eq.(4), $\cos\omega t = x/r$, so

$$v = r\omega\sqrt{1 - \frac{x^2}{r^2}} = r\omega\sqrt{\frac{r^2 - x^2}{r^2}} \quad \text{—— (6)}$$

$$\text{or} \quad v = \omega\sqrt{r^2 - x^2} \quad \text{—— (7)}$$

15- CHARACTERISTICS OF S.H.M. OF A MASS ATTACHED TO A SPRING

We have (from eq. 13.4 and 14.2)

$$a = -\frac{k}{m} x$$

$$\& \quad a = -\omega^2 x$$

Comparing the above equations, we get

$$\omega = \sqrt{k/m} \quad \ldots\ldots \quad (1)$$

So, the time period of the mass attached to a spring is

$$T = \frac{2\pi}{\omega}$$

$$\text{or} \quad T = 2\pi\sqrt{m/k} \quad \ldots\ldots \quad (2)$$

In the equation $x = r\cos\omega t$, putting $r = x_o$, we get

$$x = x_o \cos\omega t$$

$$\text{or} \quad \boxed{x = x_o \cos\sqrt{k/m} \times t} \quad \ldots\ldots \quad (3)$$

The instantaneous velocity,

$$v = \omega\sqrt{r^2 - x^2}$$

$$\text{or} \quad v = \sqrt{k/m}\sqrt{x_o^2 - x^2}$$

$$\text{or} \quad \boxed{v = x_o\sqrt{k/m}\sqrt{1 - x^2/x_o^2}} \quad \ldots\ldots \quad (4)$$

its maximum velocity , v_o when $x = 0$, as

$$\boxed{v_o = x_o\sqrt{k/m}} \quad \ldots\ldots \quad (5)$$

From eqs. (4) & (5), we get

$$v = v_o\sqrt{1 - x^2/x_o^2} \quad \ldots\ldots \quad (6)$$

The kinetic energy is

$$K.E. = \tfrac{1}{2} m v^2$$

$$= \tfrac{1}{2} m x_o^2 k/m (1 - x^2/x_o^2)$$

$$\text{or} \quad \boxed{K.E. = \tfrac{1}{2} k x_o^2 (1 - \frac{x^2}{x_o^2})} \quad ,\ldots\ldots(7)$$

Eq. (7) shows that the kinetic energy is maximum, when $x = 0$,

$$\boxed{(K.E.)_{max} = \tfrac{1}{2} k x_o^2} \quad \ldots\ldots \quad (8)$$

the kinetic energy is minimum, when $x = x_o$, (from eq.(7),

$$\boxed{(K.E.)_{min.} = 0} \quad \ldots\ldots \quad (9)$$

To calculate P.E., we have

$$F = k\,x$$

$$\text{for } x = 0,\ F = 0$$

$$\text{for } x = x,\ F = k\,x$$

So average force is

$$F_{av.} = \frac{0 + kx}{2} = \tfrac{1}{2}\,k\,x$$

Work done in displacing the mass through x is

$$\text{W. done} = \tfrac{1}{2}kx.x = \tfrac{1}{2}\,k\,x^2$$

which will appear as potential energy, so

$$\boxed{P.E. = \tfrac{1}{2}\,k\,x^2} \qquad \ldots\ldots\ (10)$$

It shows that potential energy is maximum when $x = x_o$,

$$\boxed{(P.E.)_{max} = \tfrac{1}{2}\,k\,x_o^2} \qquad \ldots\ldots\ (11)$$

The potential energy is minimum, when $x = 0$,

$$\boxed{(P.E.)_{min} = 0} \qquad \ldots\ldots\ (12)$$

Total energy at any displacement x is

$$E = PE + KE$$

$$= \tfrac{1}{2}kx^2 + \tfrac{1}{2}kx_o^2\left(1 - \frac{x^2}{x_o^2}\right)$$

$$\text{or} \quad \boxed{E = \tfrac{1}{2}\,k\,x_o^2} \qquad \ldots\ldots\ (13)$$

From eqs. (8), (11) & (13), we see that :

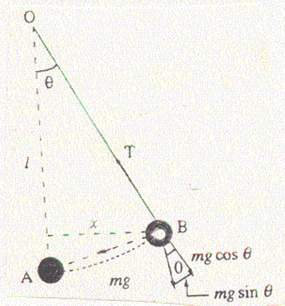
The energy oscillates back and forth between kinetic energy and potential energy but total energy of the mass remains constant everywhere.

16- THE SIMPLE PENDULUM

We define:

"A simple pendulum consists of a single isolated particle suspended from a frictionless support by a light, inextensible string".

When a simple pendulum is disturbed from its mean position, it perform a vibratory motion.



To show that

The motion of the bob is simple harmonic,

Let the bob is at position B during its vibratory motion. Two forces are acting on the bob.

i) Weight mg of the bob in vertically downward direction

ii) Tension T acting along the string

mg is resolved into two components

Component of mg along the string = $mg \cos\theta$ .....(1)

" " " perpendicular " " = $mg \sin\theta$ .....(2)

Since there is no motion along the string, so

$$T = mg\cos\theta$$

We have

$$F = m\,a \qquad \text{......}\ (3)$$

The component $mg \sin \theta$ is responsible for the motion, <u>directed towards the mean position</u>, so from eqs. (2) & (3), we get

$$\not{m}\,a = -\not{m}g\sin\theta$$

$$a = -g\sin\theta$$

$$\left[\begin{aligned} S &= r\theta \\ \text{or } x &= l\,\theta \\ \text{or } \theta &= x/l \end{aligned}\right.$$

We suppose that angle θ is very small,

so

$$\sin\theta = \theta,$$

$$a = -g\,\theta = -g\,\frac{x}{l}$$

Taylor's series expansion for,

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \text{.....}$$

$$\text{or}\quad a = -\frac{g}{l}\,x \qquad \text{.......}\ (4)$$

From eq.(4) we see that the acceleration is proportional to the displacement and directed towards the mean position, so the motion of the bob of simple pendulum execute <u>Simple Harmonic motion</u>.

To calculate time period of simple pendulum,

We have from SHM ,

$$a = -\omega^2 x \qquad \text{........ (5)}$$

Comparing eqs. (4) & (5), we get

$$\omega^2 = g/l$$

$$\text{or} \quad \omega = \sqrt{g/l} \qquad \text{........ (6)}$$

We have time period from SHM

$$T = \frac{2\pi}{\omega} \qquad \text{...... (7)}$$

From eqs. (6) & (7), we get

$$T = \frac{2\pi}{\sqrt{g/l}}$$

$$\text{or} \quad T = 2\pi\sqrt{l/g} \qquad \text{...... (8)}$$

Eq. (8) shows that the time period T of simple pendulum,

i) is independent of the mass
ii) depends upon the length l
iii) depends on the value of g

By determining T and l we can accurately measure the value of g at certain place.

17- TRANSVERSE STATIONARY WAVES IN A STRETCHED STRING

We define

<u>Stationary Waves</u>:

"Waves apparently standing still resulting from two similar wave trains travelling in opposite directions".

<u>Transverse Waves</u>:

"A wave in which the particles of the medium vibrate at right angles to the direction of travel of the wave".

To make

Formula for velocity v general formulas for wavelength λ and frequency f of transverse stationary waves.

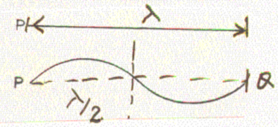


Fig.(a)

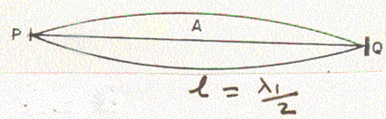


Fig.(b)

Consider a string of length l which is kept stretched at two ends so that tension in string is T.

In fig.(b), string is plucked at the middle, the string vibrates in one loop, with a frequency, say f_1, so

$$l = \frac{\lambda_1}{2} \quad \text{and} \quad v = f_1\lambda_1$$

$$\text{or } \lambda_1 = 2l \quad \text{or} \quad v = f_1 \times 2l$$

$$\text{or } \lambda_1 = \frac{2l}{1} \;..(1) \text{ or } f_1 = \frac{v}{2l} \;..(2)$$

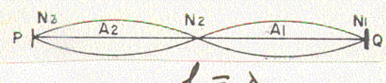


Fig.(c)

In fig.(c), string is plucked from one quarter, then it vibrates in two loops, with a frequency, say f_2, so

$$l = \lambda_2 \quad \text{and } v = f_2\lambda_2 = f_2 l$$

$$\text{or } \lambda_2 = l \quad \text{or } f_2 = \frac{v}{l} = 2\,\frac{v}{2l}$$

$$\text{or } \lambda_2 = \frac{2l}{2} \;..(3)$$

From eq.(2), we get,

$$f_2 = 2f_1 \;..(4)$$

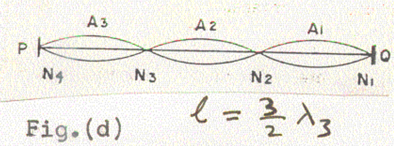


Fig.(d)

In fig.(d), the string is plucked in such a way that it vibrates in three loops, with a frequency, say f_3, so

$$l = \frac{3}{2}\lambda_3 \quad \text{and } v = f_3\lambda_3 = f_3\frac{2l}{3}$$

$$\text{or } \lambda_3 = \frac{2l}{3} \quad ..(5) \quad \text{or } f_3 = 3\frac{v}{2l} = 3f_1 \quad ..(6)$$

Rewrting eqs.(1),(3) & (5) And (2), (4) & (6)

$$\lambda_1 = \frac{2l}{1} \qquad \text{And} \qquad f_1 = \frac{v}{2l}$$

$$\lambda_2 = \frac{2l}{2} \qquad\qquad f_2 = 2f_1$$

$$\lambda_3 = \frac{2l}{3} \qquad\qquad f_3 = 3f_1$$

Generalizing the above eqs., we get

$$\boxed{\lambda_n = \frac{2}{n}l} \quad ......(7) \quad \text{And} \quad \boxed{f_n = nf_1} \quad ..... (8)$$

Now if m' is the total mass of the string, tension T and length l, then from eq. (9), we have

$$v = \sqrt{\frac{T \times l}{m'}} \quad ..... (10)$$

Putting this value in eq.(2), we get

$$\boxed{f_1 = \frac{1}{2l}\sqrt{\frac{T \times l}{m'}}} \quad ...(11)$$

From eq.(8) we conclude that we can have only quantized frequencies on the stretched string. i.e. f_1, $2f_1$, $3f_1$...... f_1 is called fundamental and others are called harmonics.

We have

$$v^2 = v \times v \qquad \left[\begin{array}{l} S = vt \\ \text{or } v = S/t = \frac{l}{t} \end{array}\right.$$

$$= v \times \frac{l}{t}$$

$$\text{or } v^2 = \frac{v}{t} \times l$$

$$= a \times l \qquad \left[a = \frac{v}{t}\right.$$

$$\text{or} \quad = \frac{T}{m} \times l \qquad \left[\begin{array}{l} F = ma \\ \text{or } a = \frac{F}{m} = \frac{T}{m} \end{array}\right.$$

$$\text{or } v^2 = \frac{T \times l}{m}$$

$$\text{so } v = \sqrt{\frac{T \times l}{m}} \quad .... (9)$$

18- NEWTON'S FORMULA FOR THE VELOCITY OF SOUND IN FLUIDS

We define:

Sound:

"The series of disturbances in matter to which the human ear is sensitive".

The velocity of sound waves depends upon the density, ρ of the medium.

Also it depends upon the elasticity, E of the medium.

Following is

Newton's formula for the velocity of sound in fluids, i.e., in liquids & gases.

"Velocity of sound is directly proportional to the square root of the elasticity and inversely proportional to the square root of the density of the medium". Mathematically,

$$v = \sqrt{\frac{E}{\rho}} \quad \ldots\ldots \quad (1)$$

We define:

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{or} \quad \rho = \frac{m}{V}$$

Elasticity(E): The property of a material body to regain its original condition, on the removal of deforming forces.

Bulk modulus:

Elasticity of volume.

Young's modulus:

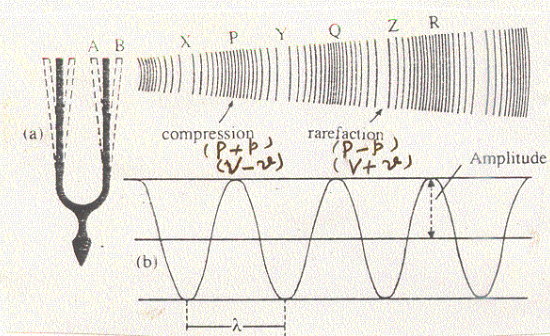
Elasticity of length.

Rigidity modulus:

Elasticity of shape.

Stress: The distorting force per unit area set up inside the body.

Strain: The change produced in the dimensions of a body under a system of forces.



Newton assumed that sound waves travel through gases in such a condition that there is no change in temperature (isothermal).

Isothermal process:

The process in which the temperature of the system remains constant.

To prove:

Elasticity of volume, E is equal to pressure, P,

Consider the volume V of the air at a pressure P.

For constant temperature, if we increase pressure from P to P + p, the volume will decrease from V to V - v, we have from Boyle's Law,

$$P_1V_1 = P_2V_2$$

$$\text{or} \quad PV = (P + p)(V - v)$$

$$\text{or} \quad \cancel{PV} = \cancel{PV} - Pv + pV + pv$$

Neglecting pv as $pv \ll P$ & V, we get

$$Pv = pV$$

$$\text{or} \quad P = p\frac{V}{v}$$

$$\text{or} \quad P = \frac{p}{v/V} = \frac{\text{stress}}{\text{strain}} = E_i$$

$$\text{so} \quad P = E \quad \ldots\ldots\ldots \quad (2)$$

From eqs. (1) and (2), we get

$$v = \sqrt{\frac{P}{\rho}} \quad \ldots\ldots\ldots \quad (3)$$

Boyle's Law:

The volume of a given mass of a gas is inversely proportional to the pressure, if the temperature is kept constant.

$$p \propto \frac{1}{V}$$

or pV = const.

There is difference of 16 % in the theoretical value of velocity of sound in air determined from the above formula and the experimental value.

Laplace's correction:

"In calculations of the velocity of sound, to use coefficient of adiabatic and not to use isothermal elasticity".

Sound waves move as longitudinal waves. They accompanied by compressions and rarefactions. At a compression the temperature of air rises and at a rarefaction temperature decreases. So constant temperature does not maintain and Boyle's law is not applicable. Instead of PV = constant, we have

$$PV^{\gamma} = \text{constant} \quad \ldots\ldots (4)$$

Adiabatic Process:

A process in which no heat flows into or out of the system.

Specific heat at constant pressure, C_p:

It is the amount of heat energy required to raise the temperature of one mole of a gas through 1°K at const.pressure.

If we increase pressure from P to P + p volume will decrease from V to V - v, so,

$$PV^{\gamma} = (P + p)(V - v)^{\gamma}$$

or $$PV^{\gamma} = (P + p)V^{\gamma}\left(1 - \frac{v}{V}\right)^{\gamma}$$

or $$P = (P + p)\left(1 - \frac{v}{V}\right)^{\gamma}$$

From Binomial theorem, we get

$$P = (P + p)\left(1 - \gamma\frac{v}{V} - \frac{\gamma(\gamma - 1)}{1.2}\frac{v^2}{V^2} - \ldots\ldots\right)$$

Neglecting squares and higher powers of (v/V) as $v << V$, we get

$$P = (P + p)\left(1 - \gamma\frac{v}{V}\right)$$

or $$P = P - P\gamma\frac{v}{V} + p - p\gamma\frac{v}{V}$$

Neglecting $p\gamma\frac{v}{V}$ as $pv << P$ & V,

$$p = P\gamma\frac{v}{V}$$

or $$\gamma P = \frac{p}{v/V} = \frac{\text{stress}}{\text{strain}} = E \quad \ldots(5)$$

From eqs. (1) & (5), we get

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \ldots\ldots (6)$$

which is Laplace's modified expression for the velocity of sound.

If we put the values in the above formula, the theoretical value agrees with the experimental value.

So Laplace's correction is correct.

<u>Specific heat at constant volume, C_v:</u>

It is the amount of heat energy required to raise the temperature of one mole of a gas through 1°K at constant volume.

We define:
$$\gamma = \frac{C_p}{C_v}$$

We have

Binomial series expansion:

$$(1 + x)^n = 1 + \frac{n}{1}x + \frac{n(n - 1)}{1.2}x^2 + \frac{n(n - 1)(n - 2)}{1.2.3}x^3 + \ldots$$

To prove:
$$PV^{\gamma} = \text{const.}$$

If we have one mole of a gas, then for adiabatic process, we have

$$Q_p = nC_v\Delta T + P\Delta V \quad \ldots(1)$$

for small change per unit vol.

$$dQ = C_v dT + P\,dV$$

for adiabatic change, we have

$$dQ = 0 = C_v dT + P\,dV$$

or $$C_v dT + P\,dV = 0 \ldots(2)$$

Now we have for one mole;

$$PV = RT$$

differentiating it, we get

$$P\,dV + V\,dP = R\,dT$$

or $$dT = \frac{P\,dV + V\,dP}{R}$$

or $$dT = \frac{P\,dV + V\,dP}{C_p - C_v} \quad \ldots.(3)$$

The speed of sound varies with the temperature of the medium.

$$v \propto \sqrt{T}$$

or more exactly

$$v = \sqrt{T}$$

$$\text{or} \quad v_1 = \sqrt{T_1} \quad \ldots\ldots(1)$$

$$\& \quad v_2 = \sqrt{T_2} \quad \ldots\ldots(2)$$

Dividing eq.(1) by eq.(2), we get

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \quad \ldots\ldots(7)$$

The speed of sound in air increases by .61 m/s per degree rise in temperature.

We have

For 1° rise in temp. change in v = .61 m/S

" t° " " " " " " = .61t m/S

e.g.

Speed at 0°C, v_o = 330 m/S

" " 1°C, v_1 = 330 + .61 = 330.61 m/S

" " 2°C, v_2 = 330 + 2x.61=331.2 m/S

" " t°C, v_t = 330 + tx.61=v_o+.61t

$$\text{So} \quad v_t = v_o + .61t$$

$$\text{or} \quad v_o = v_t - .61t \quad \ldots\ldots(8)$$

From eqs. (2) & (iii) we see that E and ρ are proportional to pressure, so the speed of sound is independent of the pressure.

From eqs. (2) & (3), we have

$$C_v\left(\frac{PdV + VdP}{C_p - C_v}\right) + PdV = 0$$

$$\text{or } C_vPdV + C_vVdP + C_pPdV - C_vPdV = 0$$

$$\text{or } C_vVdP + C_pPdV = 0$$

$$\text{or } VdP + \frac{C_p}{C_v}PdV = 0$$

$$\text{put } \frac{C_p}{C_v} = \gamma;$$

$$VdP + \gamma PdV = 0$$

Dividing throughout by PV,

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0$$

Integrating, we get

$$\log P + \gamma \log V = \text{const.}$$

or log(PV) = const.

or taking antilog,

$$PV^\gamma = \text{another const.}$$

So

$$PV^\gamma = \text{constant.}$$

We have

$$\rho = \frac{m}{V}$$

$$\text{or} \quad \rho \propto 1/V \quad \ldots\ldots(i)$$

And PV = const.

$$\text{or} \quad P \propto 1/V \quad \ldots\ldots(ii)$$

From eq.(i) & (ii), we get

$$P \propto \rho \quad \ldots\ldots(iii)$$

19- DOPPLER'S EFFECT

Statement:

"The change in the pitch of sound caused by the relative motion of either the source of sound or the Observer is called the Doppler effect".

Explanation:

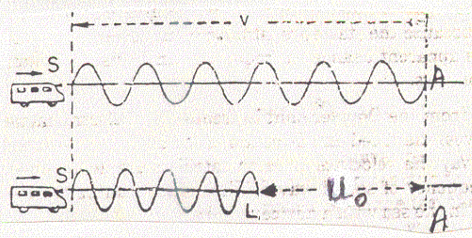
It is observed that the pitch of sound of a whistling train approaching a Observer increases and when the train is moving away the pitch decreases.

Illustration:

Consider this effect under the following cases:

1) Observer is moving towards a stationary source

When the Observer is moving towards the source with velocity $= u_o$



Now Observer receives more waves in one second than he is at rest.
So

$$\text{Additional waves} = \frac{\text{distance travelled in 1 sec.}}{\text{wavelength}} = \frac{u_o}{\lambda}$$

$v = f\lambda$ or $f = v/\lambda$

Since $v = \lambda f$ or $\lambda = \frac{v}{f}$, so

$$\text{Additional waves} = \frac{u_o}{v} f$$

And the pitch f_A of the sound heard is

$$f_A = f + \frac{u_o}{v} f = (1 + \frac{u_o}{v}) f$$

$$\text{or } \boxed{f_A = \frac{v + u_o}{v} f} \quad \ldots\ldots\ldots \quad (1)$$

As $f_A > f$, therefore the pitch of the sound heard by the Observer will increase.

2) Observer is moving away from a stationary source

If the Observer is moving away from the stationary source, the sign of u_o should be reversed, so that

$$\boxed{f_B = \frac{v - u_o}{v} f} \quad \ldots\ldots\ldots \quad (2)$$

As $f_B < f$, therefore the pitch of the sound heard by the Observer will decrease.

3) Source is moving towards the stationary Observer

Let

Position of the source be S

" " " Observer " A

frequency emitted by the source = f

velocity of the source = u_s

velocity of the sound waves = v

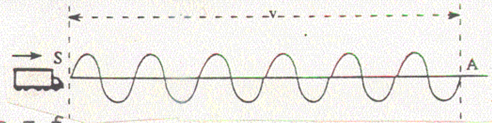


Fig.(a)

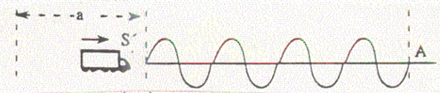


Fig.(b)

If the source is at rest, then from fig.(a), we have

$$= \frac{\text{Distance which f waves occupied}}{\text{Number of waves}}$$

No. of waves during one second is f and occupy a length v, so

$$\lambda = \frac{v}{f}$$

If the source was moving towards the Observer, shown in fig.(b), f waves emitted now contained in the length $(v - u_s)$, so

$$\lambda' = \frac{v - u_s}{f}$$

The changed frequency f_c is given by

$$f_c = \frac{v}{\lambda'} = \frac{v}{\frac{v - u_s}{f}} = \frac{v \times f}{v - u_s}$$

$$\text{or} \quad \boxed{f_c = \frac{v}{v - u_s} f} \quad \ldots\ldots\ (3)$$

As $f_c > f$, therefore the pitch of the sound heard by the Observer increases.

4.) Source is moving away from the stationary Observer

If the source is moving away from the Observer, the sign of u_s should be reversed with the result that

$$\boxed{f_D = \frac{v}{v + u_s} f} \quad \ldots\ldots\ (4)$$

As $f_D < f$, therefore the pitch of the sound heard by the Observer will decrease.

Applications of Doppler's effect:

1. Applied to light:

The frequency of light from certain stars is found to be slightly more and from other stars slightly less than the frequency of the same light emitted from the source on earth. Their velocities can be obtained from this frequency difference.

2. Ultrasonic waves from a bat:

A bat determines the location and nature of objects by sending ultrasonic waves.

3. Reflection of radar waves:

The frequency of the reflected radar waves is decreased if the plane is moving away and increased if it is approaching. From the observed frequency difference the speed and direction of the plane can be calculated.

4. Detection of submarines:

When under-water sound waves (sonar) are reflected from a moving submarine, we can detect its location.

5. Velocities of earth satellites:

These velocities are determined from the Doppler shift in the frequency of their transmitted waves.

20- YOUNG'S DOUBLE SLIT EXPERIMENT

We define:

Interference:

"The phenomenon in which the two waves support each other at some points and cancel at other".

To obtain interference of light waves, the following conditions must be fulfilled.

i) Sources must be phase coherent.

ii) Sources should be monochromatic.

iii) Linear superposition should be applicable.

We define:

Phase coherence:

Producing of two waves of same wavelength and time period at the same instant.

Monochromatic:

Light consisting of only one wavelength (or colour).

Superposition:

Combining the displacements of two or more wave motions algebraically to produce a resultant wave motion.

Cylindrical wave front:

A wavefront whose equiphase surfaces form a family of coaxial or confocal cylinders.

Young's double-slit experiment gives the experimental evidence for Huygen's wave theory of light.

The experimental arrangement is shown in fig.(1).

A screen A with slit S_o is placed in front of a monochromatic source of light.

The cylindrical wavefronts emerge on the other side of screen A.

These wave fronts arrive at screen B, which has two slits S_1 and S_2.

S_1 and S_2 behave as coherent sources. These wavefronts produce interference. The resulting interference pattern is obtained on the screen, consisting of alternate bright and dark parallel bands called fringes.

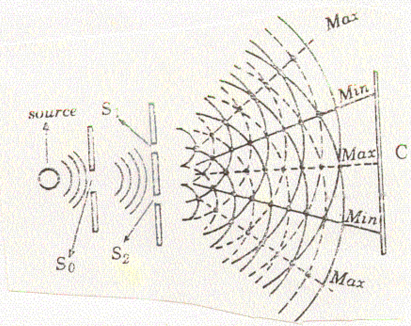


Fig.(1)

To obtain

Qualitative description of Young's experiment, see fig.(2).

Consider a point P on the screen. The waves reaching at P have distances S_1P and S_2P.

The path difference is

$$= S_2P - S_1P \quad .....(1)$$

S_1Q is drawn perpendicular to S_2P
D is distance between screen & slits
& d is distance between two slits.

since $D \gg d$

so $S_1P \simeq QP$

Also $S_2Q = d \sin\theta^*$ .....(2)

From eqs.(1) & (2), we get

$$S_2P - S_1P = d \sin\theta \quad .....(3)$$

for P to be bright fringe,
i.e. for constructive interference

$$d \sin\theta = m\lambda \quad .....(4)$$

And for dark fringes,

$$d \sin\theta = (m + \tfrac{1}{2})\lambda \quad .....(5)$$

$$m = 0,1,2,....$$

Now from fig.(2),

$$\tan\theta_m = \frac{OP}{OR} = \frac{y_m}{D}$$

or $y_m = D \tan\theta_m$

or $y_m = D \sin\theta_m$ .....(6)

since $\theta_m = \theta^*$,

so $y_m = D \sin\theta$

from eq.(4), we have

$$\sin\theta = m\lambda/d$$

So $y_m = \frac{Dm\lambda}{d} = m\lambda\frac{D}{d}$ ....(7)

or $\lambda = \frac{y_m}{m}\frac{d}{D}$ .....(8)

From eq.(8) we can calculate λ.

Constructive interference:

The interference of two waves, so that they reinforce one another. Its condition is

path difference,d = m ,
m = 0,1,2,3,...

Destructive interference:

The interference of two waves, so that they cancel one another. Its condition is

path difference,d =(m + ½) ,
m = 0,1,2,3,....

*Angle θ between any two lines is equal to the angle between their perpendiculars.

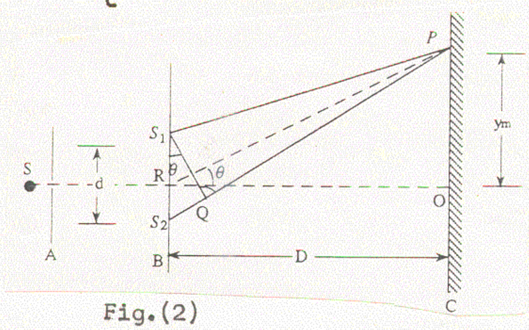


Fig.(2)

since $OP \ll OR$
so $OR \simeq PR$
or $\tan\theta_m \simeq \sin\theta_m$

Eq.(7) is: Position of mth bright fringe = $y_m = m\lambda\frac{D}{d}$ .....(9)

i.e. 0, $\lambda D/d$, $2\lambda D/d$,...

Similarly : Position of mth dark fringe = y_m = $(m + \tfrac{1}{2})\lambda D/d$ ...(10)

And Fringe width = $y_m - y_{m-1} = \frac{m\lambda D}{d} - \frac{(m-1)\lambda D}{d}$ i.e.

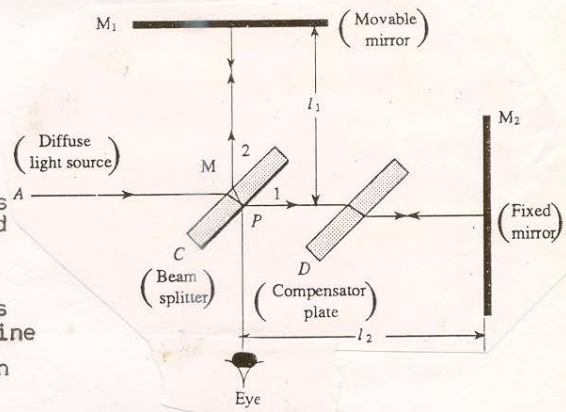
so Fringe width = $\lambda\frac{D}{d}$ .....(11)

21- THE MICHELSON INTERFEROMETER

Michelson Interferometer:

Device includes one half sivered mirror and two plane mirrors, using interference of light waves to measure very small distances.

This device splits a light beam into two parts and then recombines them to form an interference pattern. It is used for accurate measurement of wavelength.



The experimental arrangement is shown in the figure.

Monochromatic beam of light is split into two rays through half silvered mirror M.

One ray is reflected towards M_1 and second ray is transmitted through M towards mirror M_2.

After reflecting from mirrors M_1 and M_2, the two rays recombine to produce an interference, seen through a telescope.

The glass plate D is placed to compensate the path length. The path difference is varied through move-able mirror M_1.

So we see a series of bright and dark fringes.

If M_1 is moved a distance of $\lambda/4$, the path difference changes by $\lambda/2$. Then the two rays interfere constructively giving rise a bright fringe.

When M_1 is moved further $\lambda/4$, the total distance covered is $\lambda/2$, a dark fringe will appear.

Thus we see successive bright and dark fringes, as the mirror M_1 moved a distance $\lambda/4$.

The wavelength λ is measured by counting the fringe shifts m for a given displacement p of the mirror M_1. So we have

$$p = \frac{1}{2} m \lambda \qquad \text{.........} \quad (1)$$

This interference is used to make very accurate measurements.

22- POLARIZATION OF LIGHT WAVES

We define:

Polarization (of light):

"The limiting of the vibrations of light, usually to vibrations in one plane".

The phenomena of interference and diffraction proves the wave nature of light, but polarization shows that light moves as transverse waves.

To distinguish between a transverse wave and longitudinal wave, a mechanical experiment can be performed as illustrated in fig.(1).

Transverse wave on a string is passed through a wooden piece with a slit P. If the slit is at right angles, the wave is not passed. If the wave was longitudinal, the slit position does not count.

Consider a beam of ordinary light, consisting of different planes of vibrations. Also directions of vibrations are perpendicular to the propagation of waves.

Shown in fig. (3). Under certain arrangement the vibrations are allowed to pass parallel to slit. The resulting light is said to be polarized.

In light waves, a tourmaline crystal plays the same role as the wooden slit in the above mechanical illustration.

When two tourmaline crystals placed with their crystal axes parallel, a beam of light falls on them is transmitted.

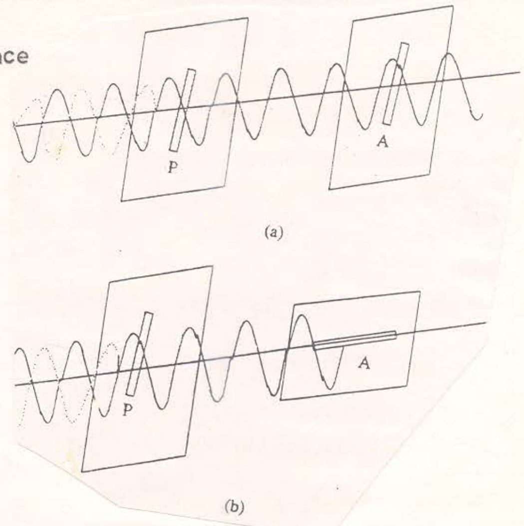


Fig. (1)

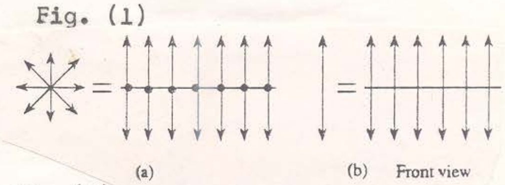


Fig.(2)

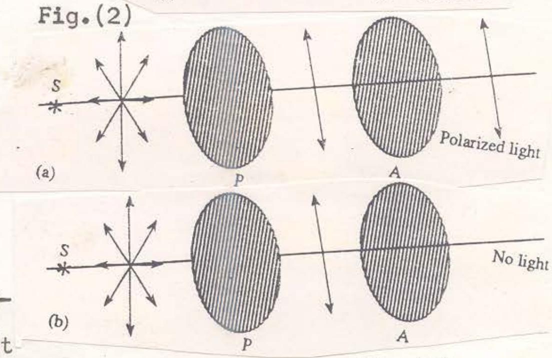


Fig.(3)

If one of them is rotated, the intensity of the transmitted light decreases and finally cut off when the axes of two crystals become perpendicular to each other.(fig. (3)). On further rotation the light reappears.

This transmitted light is called plane polarized, which is defined as a beam of light in which all the vibrations are in one direction.

Factors:

According to electromagnetic theory, light waves consists of electric and magnetic field components perpendicular to each other. When light passes through certain crystals, the electric vibrations are confined in a particular plane are moved in a single direction. In general polarization depends upon,

1. Selective absorption of light
2. Reflection of light
3. Refraction of ligh
4. Scattering of ligh.

APPLICATIONS:

1. Polaroid filters:

   Its a transparent plastic sheet in which needle like crystals are embeded. These filters are used in many fields for polarization of light.

2. Optical activity:

   Concentration of sugar in blood or urine is determined through polarized light.

3. Curtainless window:

   An outer polarizing disc is fixed and an inner one is rotated to adjust the amount of light.

4. Head lights:

   At night head-light glare can be controlled through polarizing headlights and light polarizing viewer.

5. Photography :

   Polarizing discs are used in front of camera lens to enhance the effect of sky.

23- Equation of Continuity

STATEMENT:

"The product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is a constant". Mathematically,

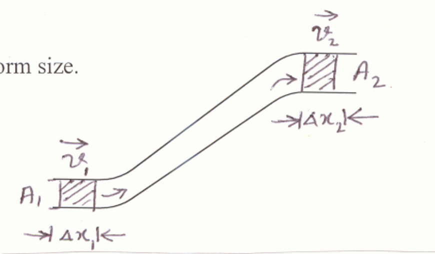
$$A_1 v_1 = A_2 v_2$$

PROOF:

Consider

A fluid flowing through a pipe of non-uniform size.

And the flow of the liquid is streamline & incompressible.



Let

As in the figure

At left side:

Velocity of the fluid = v_1

Move through distance = Δx_1

Area of cross-section = A_1

So volume = $V_1 = \Delta x_1 . A_1$

& mass passing during Δt

Density = mass / volume

$\rho\, m / V$

or $m = \rho V$

$$\Delta m_1 = \rho_1 V_1 = \rho_1 \Delta x_1 . A_1$$

or $\Delta m_1 = \rho_1 A_1 v_1 . \Delta t$ ..... (1)

$S = vt$

or $\Delta x_1 = v_1 t$

At right side:

Velocity of the fluid = v_2

Move through distance = Δx_2

Area of cross-section = A_2

So volume = $V_2 = \Delta x_2 . A_2$

& mass passing during Δt

$$\Delta m_2 = \rho_2 V_2 = \rho_2 \Delta x_2 . A_2$$

or $\Delta m_2 = \rho_2 A_2 v_2 . \Delta t$ ..... (2)

As the streamline flow is incompressible, so

$\Delta m_1 = \Delta m_2$ .....(3)

from equations (1), (2) & (3) we have

$$\rho_1 A_1 v_1 . \Delta t = \rho_2 A_2 v_2 . \Delta t$$

since density is constant, i.e., $\rho_1 = \rho_2 = \rho$, so

$$\rho A_1 v_1 = \rho A_2 v_2$$

$$A_1 v_1 = A_2 v_2$$

Which is Equation of Continuity.

24- Bernoulli's Equation

STATEMENT:

In a steady frictionless motion of a fluid acted on by external forces which possess a gravitational potential ρgh, then

$$P + \frac{1}{2}\rho v + \rho gh = \text{constant}$$

where P & ρ are the pressure and density of the fluid, v is the velocity of the fluid along a streamline.

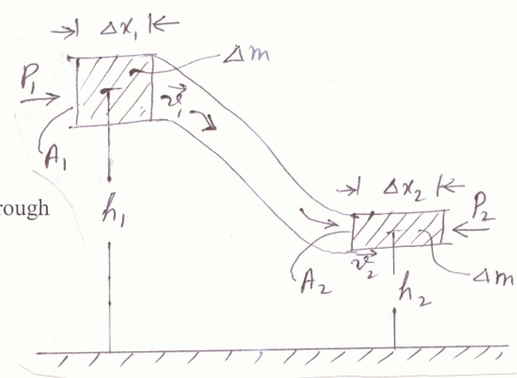
PROOF:

Consider

A fluid is flowing , (in the figure)

And assume;

1) The fluid is incompressible,
2) Non-viscous,
3) Moving with streamline flow



Let (shown in the figure)

A liquid of mass (Δm), flowing through A pipe during time (t),

At left side:

Pressure = P_1
Velocity of the fluid = v_1
Move through distance = Δx_1
Area of cross-section = A_1
Height from the bottom = h_1

At right side:

(for the same mass Δm)
Pressure = P_2
Velocity of the fluid = v_2
Move through distance = Δx_2
Area of cross-section = A_2
Height from the bottom = h_2

We have

Pressure = P = Force / Area = F / A or F = P A ..... (1)

Work done = W = force x displacement = F x Δx = PA Δx ..... (2)

Also S = Δx = v t .....(3)

& $\rho = m / V$ or $V = m / \rho$

as volume = area x length

so A . Δx = A v t = V = m / ρ .....(4)

for the same mass flowing during time t, through both ends, the volume will be

$A_1 v_1 t = A_2 v_2 t = A v t$ .....(5)

Now from equations (2) & (4) we have

W = P A v t

Or W = P m / ρ .....(6)

Now we have

Kinetic energy = KE = $\frac{1}{2} m v^2$ .....(7)

& Potential energy = PE = $m g h$ .....(8)

Taking mass <u>(Δm)</u> of the fluid flowing from upper end to lower end as <u>same.</u>

Applying the <u>Law of conservation of energy</u> to this volume ((Δm) of fluid:

Net Work done = change in KE + change in PE

Or $W_{upper\ end} + W_{lower\ end} = \{KE_{upper} - KE_{lower}\} + \{PE_{upper} - PE_{lower}\}$ .....(9)

From equations (6) to (9) we have

$$P_1 m / \rho + \{(-P_2) m / \rho\} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + m g h_2 - m g h_1$$

Or $m / \rho (P_1 - P_2) = m(\frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 + g h_2 - g h_1)$

Or $P_1 - P_2 = \rho(\frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 + g h_2 - g h_1)$

Or $P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 + \rho g h_2 - \rho g h_1$

Or $P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$

Or $P + \frac{1}{2} \rho v^2 + \rho g h =$ constant

Which is <u>Bernoulli's Equation</u>.

24- MICROSCOPES

a) Magnifying glass (or Simple Microscope):

We define:

"An ordinary convex lens held close to the eye is called magnifying glass".

Least distance of distinct vision,(d):

The distance equal to 25 cm for a normal person to see clearly an object. (fig. a)

Magnifying power (M) :

The ratio of the angle formed by the image of an object seen through an eye piece at the eye to the angle formed by the same object when both are placed at the least distance of distinct vision from the eye.

Mathematically,

$$M = \frac{\beta}{\alpha} \quad \ldots\ldots\ (1)$$

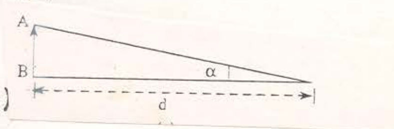


Fig. (a)

From the fig., we have

$$\tan\beta = \frac{A'B'}{d} \quad \& \quad \tan\alpha = \frac{AB}{d}$$

for small angle,

$$\tan\beta = \beta = \frac{A'B'}{d} \quad \& \quad \tan\alpha = \alpha = \frac{AB}{d}$$

$$\text{So } M = \frac{\beta}{\alpha} = \frac{A'B'/d}{AB/d} = \frac{A'B'}{AB} \quad \ldots(2)$$

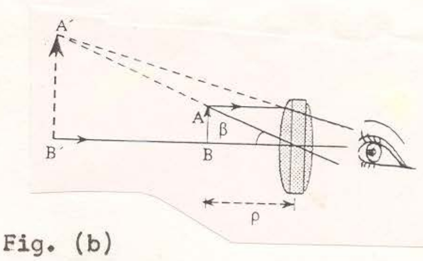


Fig. (b)

In fig.(b), triangles A'O B' and AOB are similar, so

$$\frac{A'B'}{A\,B} = \frac{d}{p} \quad \ldots\ldots\ (3)$$

From eqs. (2) & (3), we get

$$M = \frac{d}{p} \quad \ldots\ldots(4)$$

From the equation,

$$1/f = 1/p + 1/q$$

since $q = -d$, as virtual image,

so $1/f = 1/p - 1/d$

multiplying both sides by d, we get

$$d/f = d/p - 1$$

$$\text{or } d/p = 1 + d/f \quad \ldots(5)$$

Taylor's series for,

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots$$

for small θ,

$$\sin\theta = \theta$$

$$\cos\theta = 1$$

so $$\tan\theta = \frac{\sin\theta}{\cos\theta} = \theta$$

From equations (4) and (5), we get

$$M = 1 + \frac{d}{f} \quad ....(6)$$

So magnifying power of magnifying glass is inversely proportional to f. Lesser the focal length, greater will be its magnification.

b) Compound Microscope :

We define:

"Compound microscope is a device used to produce a very large magnification of very small objects. It consists of an objective and an eye-piece".

Construction:

Compound microscope (fig.c)

Consists of two convex lenses. ----an objective of short focal length and small aperture and eye-piece of large focal length and large aperture as compared to the objective.

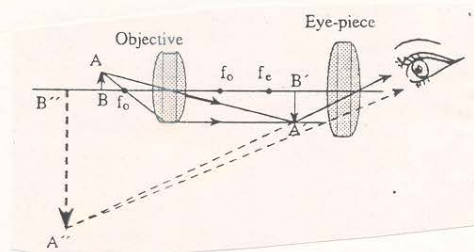


Fig. (c)

Working :

The object AB forms a real, inverted and enlarged image A'B' of the object placed just beyond the focus of the objective.

The eye piece is used as a magnifying glass to see the final image A''B'' at least distance of distint vision, d. It is virtual and very much enlarged.

Magnifying power:

We define:

$$\text{Magnifying power} = \frac{\text{Angle formed by final image}}{\text{Angle formed at naked eye}}$$

$$\text{or} \quad M = \frac{\beta}{\alpha}$$

From the figure, we have

$$\tan\beta = \beta = \frac{A''B''}{d}$$

and (from fig. a), $\tan\alpha = \alpha = \frac{AB}{d}$

$$\text{So} \quad M = \frac{\beta}{\alpha} = \frac{\frac{A''B''}{d}}{\frac{AB}{d}} = \frac{A''B''}{AB}$$

$$\text{or} \quad M = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB} = M_1 \times M_2 \quad ....(7)$$

In eq. (7), M_1 is the magnification produced by the eyepiece and M_2 that produced by the objective.

Now, in the fig.(c), triangles A'O B' and A O B are similar, so

$$\frac{A'B'}{AB} = \frac{B'O}{BO}$$

or

$$\frac{A'B'}{AB} = \frac{q}{p} \quad \ldots\ldots(8)$$

which is the magnification produce by objective.

Now, magnification produced by the eye-piece (see eq.8),

$$M_1 = \frac{A''B''}{A'B'} = 1 + \frac{d}{f_e} \quad \ldots\ldots(9)$$

From eqs.(7), (8) & (9), we get [$f = f_e$ = focal length of eye-piece.]

$$M = \frac{q}{p}\left(1 + \frac{d}{f_e}\right) \quad \ldots\ldots \; (10)$$

Usually, the object AB lies very close to the focus of the objective of focal length f_o, so

$$f_o = p \quad \ldots\ldots\ldots \; (11)$$

And image A'B' lies very close to the eye-piece and image distance q is approximately equal to the length L of the microscope tube, so

$$q = L \quad \ldots\ldots\ldots \; (12)$$

From eqs. (10), (11) & (12), we get

$$\boxed{M = \frac{L}{f_o}\left(1 + \frac{d}{f_e}\right)} \quad \ldots\ldots\ldots \; (13)$$

$f_o < f_e$

which is required formula for magnification of compound microscope. From here we see that for high magnification the objective and eye-piece should be of short focal length. However, $f_o < f_e$.

25- TELESCOPES

We define:

Telescope :

"A device for collecting and producing an image of distant objects".

To see distant objects (e.g. distant galaxies) more amount of light is needed. So the objective lens used in a telescope is of large focal length with large aperture.

There are two types of telescopes:

a) Reflecting telescope:

An instrument which uses a concave mirror to bring light of distant objects to a focus.

b) Refracting telescope:

An instrument which uses a lens to bring light of distant objects to a focus.

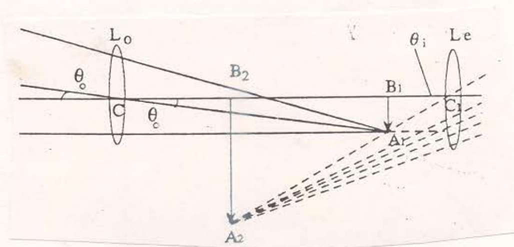
Three types of refracting telescopes will be discussed below.

1. Astronomical Telescope:

We define:

"It is a telescope used to see heavenly bodies; it consists of two convex lenses, one for objective and the other as an eye-piece".

Details:



The objective is a convex lens. It has large focal length and large aperture. To reduce chromatic and spherical aberrations, usually combination of two lenses in contact is used.

The eye-piece is also a convex lens. It has short focal length and small aperture. To reduce chromatic and spherical aberrations, combination of two lenses separated by a suitable distance is used.

The objective is mounted at one end of a tube and eye-piece is mounted in a small tube to slide inside the bigger tube of the objective.

Working:

The objective form a real, inverted and diminished image at its focus B_1 of a distant object, in front of eye piece.

The distance between the eye-piece and this image is adjusted within the focal length so that a magnified and virtual image is formed at the least distance of distinct vision. If the image A_1B_1 is made at the focus of eye-piece then the final image is formed at infinity. It is called the telescope is focused for infinity. Then

$$\text{Length of the telescope} = f_o + f_e \quad \ldots\ldots (1)$$

where f_o = focal length of objective,
f_e = " " " eye-piece.

Here the final image is formed inverted, which makes no difference for astronomical purposes.

<u>Magnifying power</u> :

We define:

"It is the ratio of the angle formed by the image at the eye as seen through the telescope to the angle formed by the object with unaided eye, the object and image both lying at infinity".

Mathematically,
$$M = \frac{\theta_i}{\theta_o} \quad \ldots\ldots\ldots (2)$$

In the figure,
$$\angle A_1 C B_1 = \theta_o \quad \ldots\ldots\ldots (3)$$
$$\& \quad \angle A_2 C_1 B_2 = \angle A_1 C_1 B_1 = \theta_i \quad \ldots\ldots (4)$$

Now for small angles
$$\theta_o = \tan\theta_o = \frac{A_1B_1}{B_1C} \quad \ldots(5)$$
$$\& \quad \theta_i = \tan\theta_i = \frac{A_1B_1}{B_1C_1} \quad \ldots\ldots(6)$$

From eqs. (2) to (6), we get
$$M = \frac{A_1B_1/B_1C_1}{A_1B_1/B_1C} = \frac{A_1B_1}{B_1C_1} \times \frac{B_1C}{A_1B_1}$$
$$\text{or} \quad M = \frac{B_1C}{B_1C_1} \quad \ldots\ldots (7)$$

Taylor's series for,
$$\sin\theta = \theta - \frac{\theta}{3!} + \frac{\theta}{5!} - \cdots\cdot$$
$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots\cdot$$

for small θ,
$$\sin\theta = \theta$$
$$\cos\theta = 1$$
$$\text{so} \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \theta$$

When telescope is focused for infinity,
$$B_1C = \text{focal length of the objective} = f_o \quad \ldots.(8)$$
$$\& \quad B_1C_1 = \text{" " " " eye-piece} = f_e \quad \ldots.(9)$$

From eqs. (7), (8) & (9), we get
$$\boxed{M = \frac{f_o}{f_e}}, \quad f_e < f_o \quad \ldots. (10)$$

Postulates of the Kinetic Theory of Gases

The kinetic-molecular theory of gases is based on the following main assumptions first stated by Clausius.

1. A chemically uniform gas consists of very small identical molecules.
2. The molecules are constantly in random motion, moving in all directions with all possible velocities.
3. The molecules behave like smooth elastic spheres.
4. The energy of the gas is all kinetic.
5. The time spent in a collision is negligible as compared with that during which the molecules are moving independently.
6. Between collisions the molecules move in a straight line with uniform velocity.
7. The molecular radii are assumed to be negligibly small as compared with the mean free path.
8. The average kinetic energy of gas molecules is proportional to the absolute temperature.

27- Pressure of a gas from Kinetic Theory

Consider a cubical container of side L

Area of one side = A ,

& Volume = L.A = L^3 = V

Let a molecule is moving along X-direction,

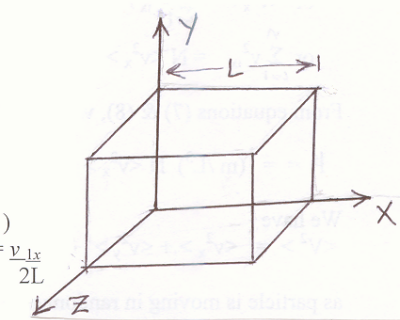
Its velocity will be = v

Time interval = t

Distance traveled = v t

Distance traveled between two consecutive collisions = 2L

[S = v t & t = S/v]



Time for one collision = $2L / v_{1x}$ ….. (1)

No. of collisions per second = $\dfrac{1}{2L / v_{1x}} = \dfrac{v_{1x}}{2L}$

No. of collisions in $\Delta t = v_{1x} \cdot \Delta t / 2L$

Momentum of the molecule before collision = $m\,v_{1x}$

Momentum of the molecule after collision = $-m\,v_{1x}$

The change in momentum of the molecule = $-m\,v_{1x} - (m\,v_{1x})$

$= -2\,m\,v_{1x}$ ….. (2)

rate of change of momentum = $\dfrac{-2\,m\,v_{1x}}{2L / m\,v_{1x}}$

$= \dfrac{-m\,v_{1x} \cdot v_{1x}}{L} = \dfrac{-m\,v_{1x}^{2}}{L}$ …..(3)

Now we have

Force = f = m a = $\dfrac{m\,(v_f - v_i)}{t}$

$= \dfrac{m\,(v_f - v_i)}{t}$ = Rate of change of momentum …..(4)

from equations (3) & (4)

$$F = \frac{-m v_{1x}^2}{L}$$

Where F is the force exerted by the wall on a molecule
So force exerted on the wall by a moleule is

$$-F = \frac{-m v_{1x}^2}{L} \quad \text{or} \quad F = \frac{m v_{1x}^2}{L} \qquad \ldots (5)$$

And total force exerted on right wall by all the molecules, F_x , will be

$$F = \frac{m v^2_{1x}}{L} + \frac{m v^2_{2x}}{L} + \frac{m v^2_{3x}}{L} + \ldots\ldots\ldots + \frac{m v^2_{nx}}{L}$$

$$\text{or} \quad F = \frac{m}{L} \Sigma v^2_{ix} \qquad \ldots (6)$$

So pressure p on the wall will be

$$p = (m/L)\, \Sigma\, m\, v^2_{ix} / L^2 \qquad [\, p = F / A \;\&\; A = L^2$$

$$\text{or} \quad p = (m/L^3)\, \Sigma\, m\, v^2_{ix} \qquad \ldots (7)$$

Now we define
mean square velocity $<v^2_x>$, as

$$<v^2_x> = \frac{v^2_{1x} + v^2_{2x} + v^2_{3x} + \ldots\ldots\ldots\ldots + v^2_{nx}}{N}$$

$$\text{or} \quad <v^2_x> = \Sigma v^2_{ix} / N$$

$$\text{or} \quad \Sigma v^2_{ix} = N <v^2_x> \qquad \ldots (8)$$

From equations (7) & (8), we have

$$P = = (m/L^3)\; N <v^2_x> \qquad \ldots (9)$$

We have

$$<V^2> = <v^2_x> + <v^2_y> + <v^2_z>$$

as particle is moving in random direction so

$$<v^2_x> = <v^2_y> = <v^2_z> = 1/3 <v^2> \qquad \ldots (10)$$

from equations (9) & (10), we have

$$p = (m N/L^3)\; 1/3 <v^2> \qquad \ldots (11)$$

$$p = (m N / V)\; 1/3 <v^2> \qquad [\, vol = V = L^3 \,]$$

$$= (N/V)\, m\; 1/3 <v^2>$$

$$= (2 N / V)\; ½\, m\; 1/3 <v^2>$$

$$p = (2/3\; N / V) <1/2\, mv^2> \qquad \ldots (12)$$

$$\text{or} \quad p = 2/3\, (N / V) <KE> \qquad \ldots (13)$$

Defining

N_o = No. of molecules / Volume = N / V
& $<1/2\ mv^2> \;=\; (KE)_{av}$ } ….. (14)

from equations (12) & (14), we have

$$p = 2/3\ N_o\ (KE)_{av} \qquad \text{….. (15)}$$

it implies that

$$p \qquad (KE)_{av} \qquad \text{….. (16)}$$

Defining temperature T :

We have from previous knowledge

$$pV = nRT$$

or $p = nRT/V$ ….. (17)

from equations (12) and (17), we have

$$nRT/V = (2/3\ N/V) <1/2\ mv^2>$$

or $T = 2/3\ (N/nR) <1/2\ mv^2>$ ….. (18)

or $T \qquad (KE)_{av}$ ….. (19)

Derivation of gas laws from kinetic theory of gases:

We have from previous knowledge [eq (12)],

$$p = (2/3\ N/V) <1/2\ mv^2> \qquad \text{….. (1)}$$

If average KE = $<1/2\ mv^2>$ is constant, then

$$pV = (2/3\ N) \times \text{constant} = \text{constant} \qquad \text{….. (2)}$$

which is Boyles's Law.

Now from eq. (1)

$$V = (2/3\ N/p) <1/2\ mv^2> \qquad \text{….. (3)}$$

If pressure p is constant, then

$$V = \text{constant} \times <1/2\ mv^2>$$

or $V \qquad (KE)_{av}$

As $(KE)_{av}$ is measure of Temperature T, so

$$V \qquad T$$

which is Charles' Law.

28- Thermal Expansion

To prove $\beta = 3\alpha$

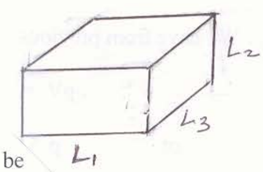
We define,

$$\beta = \frac{\Delta V}{V_o \, \Delta T}$$

or $\Delta V = \beta \, V_o \, \Delta T$ ..... (1)

Considering a rectangular parallelepiped with dimension L_1, L_2 and L_3, then

$$V_o = L_1 \, . \, L_2 \, . \, L_3$$



For linear expansion, we have

$$L = L_o (1 + \alpha \, \Delta T)$$

So the length of each side changes and the new volume will be

$$\begin{aligned}
V_o + \Delta V &= L_1 (1 + \alpha\Delta T) \times L_2 (1 + \alpha\Delta T) \times L_3 (1 + \alpha\Delta T) \\
&= L_1 L_2 L_3 \, (1 + \alpha\Delta T)^3 \\
&= V_o (1 + \alpha\Delta T)^3 \qquad [\, V_o = L_1 L_2 L_3 \,] \\
& \qquad [\, (a + b)^3 = a^3 + 3a^2 b + 3a b^2 + b^3 \,] \\
&= V_o \{1 + 3(1)^2 (\alpha \, \Delta T) + 3(1)(\alpha \, \Delta T)^2 + (\alpha \, \Delta T)^3 \} \\
&= V_o \{1 + 3\alpha \, \Delta T + 3\alpha^2 \, \Delta T^2 + \alpha^3 \, \Delta T^3 \}
\end{aligned}$$

since $\alpha \, \Delta T$ is very small, neglecting its higher powers, we get

$$V_o + \Delta V = V_o \{1 + 3\alpha \, \Delta T\}$$

or $V_o + \Delta V = V_o + V_o \times 3\alpha \, \Delta T$

or $\Delta V = V_o \times 3\alpha \, \Delta T$

or $\Delta V = 3 \, \alpha \, V_o \, \Delta T$ ..... (2)

Comparing equations (1) and (2), we get

$$\beta = 3 \, \alpha$$

which is required proof.

29- First Law of Thermodynamics

Statement

"The heat energy supplied to a system is equal to the increase in the internal energy of the system from an initial value U_i to the final value U_f plus the work done by the system on its surroundings". Mathematically

$$\Delta Q = \Delta U + \Delta W \qquad \text{..... (1)}$$

Explanation

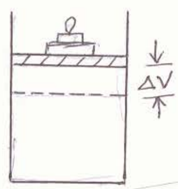
Eq. (1) defines the change in the internal energy of a system. It is equal to the energy flowing in as heat energy minus the energy flowing out as work.
The first law of thermodynamics indicates that there exists a useful state variable of every thermodynamic system called the internal energy.

Applications:

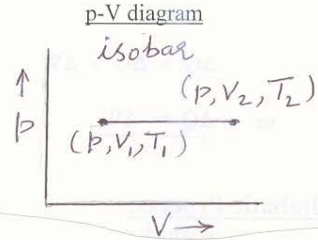
1. Isobaric Process:

"The process in which the pressure of the system remains constant".

Gas-cylinder system



p-V diagram



Applying the equation in this isobaric process:

$$\Delta Q = \Delta U + \Delta W$$

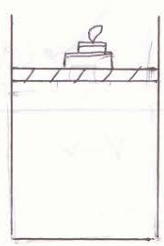
or $\Delta Q = \Delta U + p\Delta V$

Work = force x displacement
$W = F \times d$ [$p = F/A$
or $W = pAd$ [or $F = pA$
or $W = pV$ [$A \times d = V$
or $\Delta W = p\Delta V$

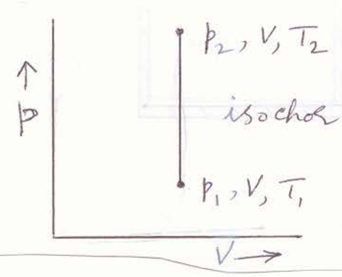
2. Isochoric Process:

"The process in which the volume of the system remains constant".

The System



p-V diagram



Applying the equation,

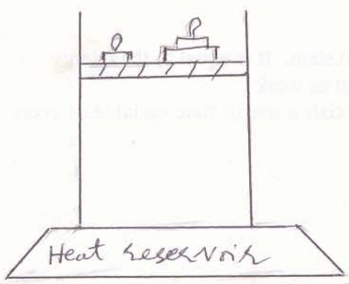
$$\Delta Q = \Delta U + \Delta W \qquad \Delta W = 0$$

or $$\Delta Q = \Delta U$$

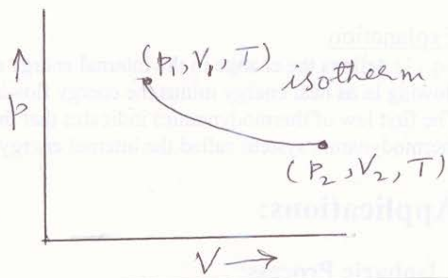
3. Isothermal Process:

"The process in which the temperature of the system remains constant".

The System



p-V diagram



Applying the equation,

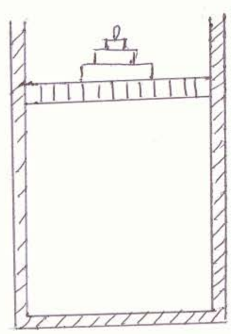
$$\Delta Q = \Delta U + \Delta W \qquad \Delta U = 0$$

or $$\Delta Q = \Delta W$$

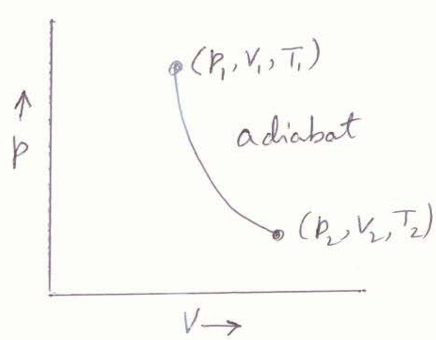
4. Adiabatic Process:

"The process in which no heat enters or leaves the system".

The System



p-V diagram



Applying the equation,

$$\Delta Q = \Delta U + \Delta W \qquad \Delta Q = 0$$
$$0 = \Delta U + \Delta W$$
$$\text{or} \quad \Delta W = -\Delta U$$

Also in adiabatic changes the following relation is found to be true,

$$p V^{\gamma} = \text{constant} \qquad \gamma = C_p / C_V$$

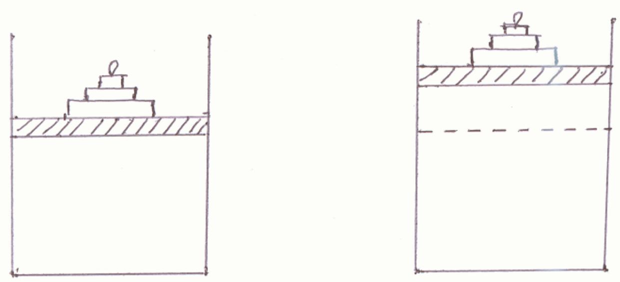
5. Heat Capacity of an Ideal Gas:

We have from previous knowledge

$$\Delta Q = mc\Delta T$$

for molecular specific heat

$$\Delta Q = nC\Delta T$$



At constant volume:

$\Delta Q = nC_V \Delta T$ = Heat energy used in raising the temperature through ΔT ….. (1)

& Heat energy used in doing the external work $= \Delta W = pAd = p\Delta V = nR\Delta T$ [$pV = nRT$] ….. (2)

At constant pressure:

$\Delta Q = nC_p \Delta T$ = Heat energy used in raising the temperature through ΔT + Heat energy used in doing the external work ….. (3)

from equations (1), (2) & (3), we get

$$nC_p \Delta T = nC_V \Delta T + nR\Delta T$$

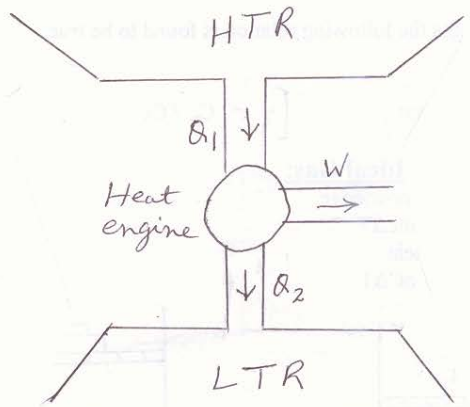
$$\text{or} \qquad C_p = C_V + R$$

It implies $\quad C_p > C_V$

30- Second Law of Thermodynamics

Lord Kelvin's Statement

"No heat engine operating continuously in a cycle, can extract heat from a heat reservoir and convert all of it into work".



Clausius Statement

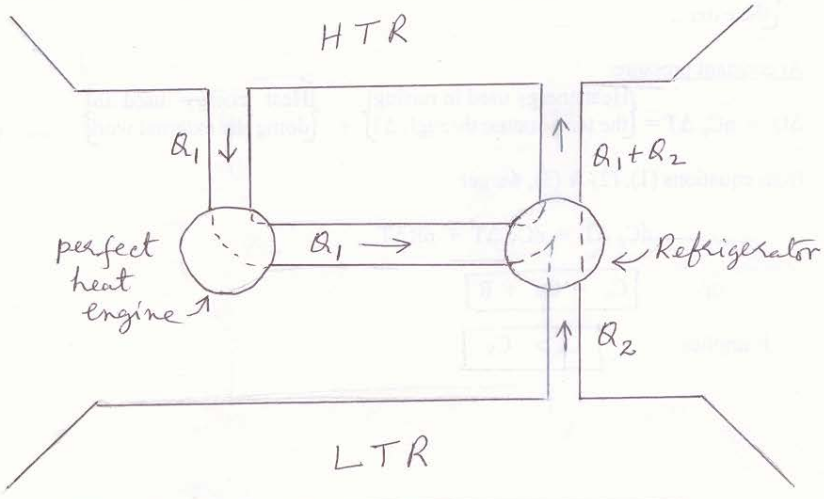
"It is impossible to cause heat to flow from a cold body to a hot body without the expenditure of energy".

BOTH STATEMENTS ARE EQUIVALENT

It can be proved by showing that, if either statement is false the other statement must be false also.

Suppose that Clausius statement were false so that we could have a refrigerator operating without doing any work on it. We could use an ordinary engine to remove heat from a hot body, to do work and to return part of the heat to a cold body.

By connecting our perfect refrigerator into the system, this heat would be returned to the hot body without the expenditure of any work. It violates the Kelvin's statement. If we reverse this reasoning, even then the net result is a transfer of heat from cold to hot body without expenditure of work. This is the violation of Clausius statement.



31- Entropy & Second Law of Thermodynamics

Entropy:

"The physical quantity which describes the ability of a system to do work and it also describes disorder of a system". Mathematically

$$\Delta S = \Delta Q / T$$

Entropy is a state variable. It is a measure of disorder. The more disordered the state of a system, the larger will be its entropy.

Another form of Second Law of Thermodynamics

"If an isolated system undergoes change, it will change in such a way that its entropy either remains constant or it tends to be maximum".

Relating the both

In the definition and application of the Second Law of Thermodynamics, Clausius was the first to introduce a new physical quantity, called entropy, which has proved to be of great importance not only in the further development of thermodynamics but also in the recognition of a fundamental law of Nature. The problem of continuous conversion of heat into work, with which the second law deals, is largely dependent on the direction rather than the actual amount of energy change in a system. We find that the new concept, entropy, can cover that additional factor. In the application of the second law, the change of the thermal state of the working substance is more important than the general idea of more convertibility of heat in work, since it is the working substance alone which undergo a thermo-dynamical change in the process, so entropy can efficiently define the thermo-dynamical state of any working substance. Also entropy deals with the physical property of a substance that can remain constant in adiabatic change.
The above arguments lead us to restate second law of thermodynamics in terms of entropy. That is, the entropy of the Universe during any process either remains constant or increases.