Core Physics

–One Hour Revision

of the Textbook

PHYSICS XI

1. First thoroughly understand & revise / memorize the article.
2. Memorize green background statements / equations of the Text Book.
3. Write a summary of main points / master equations.
4. Sample summary is given, but make your own.
5. Do not copy or follow it. It is just for your guideline.
6. Or write down central points / equations on ($4'' \times 6''$) cards.
7. Use these Cards / Core Phy for memorizing & for last minutes look.

Ross Nazir Ullah

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Chap1

Significant Figures:

Accurately known digits and the first doubtful digit.

A **precise** measurement is the one which has less absolute uncertainty.

An **accurate** measurement is one which has less fractional or percentage uncertainty.

Dimensions of Physical Quantity:

$$[F] = [m][a] = [M][LT^{-2}] = [MLT^{-2}]$$

Chap 2

Vector Addition by Rectangular Components

Consider vectors $\vec{A}$ & $\vec{B}$.
Found resultant $\vec{R}$ by head-to-tail rule.
Did some geometrical work.
From the geometry of the figure,

$$R_x = A_x + B_x \quad \& \; R_y = A_y + B_y$$

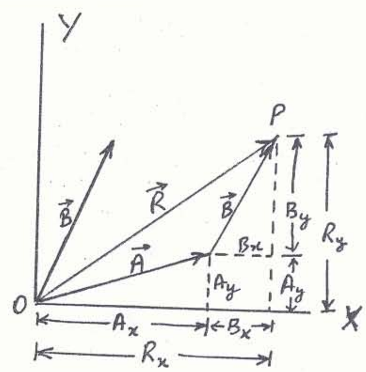
Generalizing the equations,

$$R_x = A_x + B_x + C_x + \ldots\ldots$$

$$\& \quad R_y = A_y + B_y + C_y + \ldots\ldots\ldots$$

The magnitude is, $R = \sqrt{R_x^{\,2} + R_y^{\,2}}$

& direction is, $\tan\theta = \dfrac{R_y}{R_x}$



Product of two Vectors

$\vec{A}\cdot\vec{B} = AB\cos\theta$; $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$ & $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$$\& \quad \cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$; $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

$\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$ & $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\,\hat{i} + (A_zB_x - A_xB_z)\,\hat{j} + (A_xB_y - A_yB_x)\,\hat{k}$$

Torque : $\vec{\tau} = \vec{r}\times\vec{F} = (rF\sin\theta)\,\hat{n}$

Conditions of Equilibrium

1^{st} Condition of Equilibrium, $\Sigma\vec{F} = 0$ or i) $\Sigma\vec{F}_x = 0$ & ii) $\Sigma\vec{F}_y = 0$

2^{nd} Condition of Equilibrium, $\Sigma\,\vec{\tau} = 0$

Chap 3

Law of conservation of linear momentum

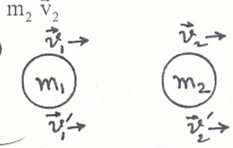
$$\text{Impulse} = \vec{F} \times t = m\,\vec{v}_f - m\,\vec{v}_i$$

For two balls: $\vec{F} \times t = m_1\,\vec{v}_1' - m_1\,\vec{v}_1$ & $\vec{F}' \times t = m_2\,\vec{v}_2' - m_2\,\vec{v}_2$

Adding, $(\vec{F} + \vec{F}')\,t = (m_1\,\vec{v}_1' - m_1\,\vec{v}_1) + (m_2\,\vec{v}_2' - m_2\,\vec{v}_2)$

As $\vec{F}' = -\vec{F}$, so $(m_1\,\vec{v}_1' - m_1\,\vec{v}_1) + (m_2\,\vec{v}_2' - m_2\,\vec{v}_2) = 0$

& $m_1\,\vec{v}_1 + m_2\,\vec{v}_2 = m_1\,\vec{v}_1' + m_2\,\vec{v}_2'$



Hence "The total linear momentum of an isolated system remains constant."

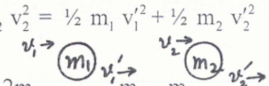
Elastic Collision in One Dimension

Consider, two masses m_1 & m_2 with initial & final velocities as v_1, v_2 and v_1' & v_2'.

From law of conservation of momentum, $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$

& from law of conservation of K.E., $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$

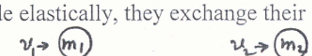
After solving for v_1' & v_2', we get



$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \quad \& \quad v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

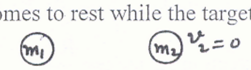
Case 1 When $m_1 = m_2$; from equations, $v_1' = v_2$ & $v_2' = v_1$

Conclusion: When two particles of equal masses collide elastically, they exchange their velocities.



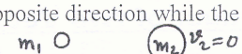
Case 2 When $m_1 = m_2$ & $v_2 = 0$; from the above equations, $v_1' = 0$ & $v_2' = v_1$

Conclusion: The incident particle which was moving with v_1, comes to rest while the target particle that was at rest begins to move with velocity v_1.



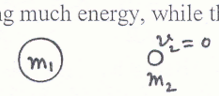
Case 3 When $m_2 \gg m_1$ & $v_2 = 0$; from the above equations, $v_1' \simeq -v_1$ & $v_2' \simeq 0$

Conclusion: The small incident particle just bounces off in the opposite direction while the heavy target remains almost motionless.



Case 4 When $m_1 \gg m_2$ & $v_2 = 0$; from the above equations, $v_1' \simeq v_1$ & $v_2' \simeq 2v_1$

Conclusion: The incident particle keeps on moving without loosing much energy, while the target particle moves with the double velocity.



Projectile

"**Projectile motion** is two dimensional motion under constant acceleration due to gravity"

Maximum height h: $2aS = v_f^2 - v_i^2$ or $2(-g)h = (0)^2 - (v_i \sin\theta)^2$

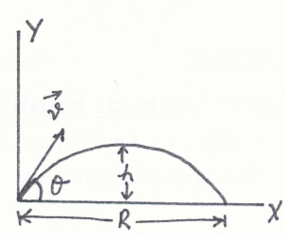
or $-2gh = -v_i^2 \sin^2\theta \Rightarrow h = \dfrac{v_i^2 \sin^2\theta}{2g}$

Time of flight t: $S = v_i t + \frac{1}{2} a t^2$ or $0 = v_i \sin\theta\, t + \frac{1}{2}(-g) t^2$

or $0 = v_i \sin\theta\, t - \frac{1}{2} g t^2 \Rightarrow t = \dfrac{2v_i \sin\theta}{g}$

Range R: $S = v\,t$ or $R = v_i \cos\theta \dfrac{2v_i \sin\theta}{g}$

or $R = \dfrac{v_i^2}{g} 2\sin\theta\cos\theta \Rightarrow R = \dfrac{v_i^2}{g}\sin 2\theta$



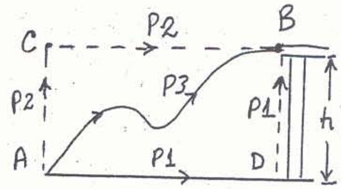
An un-powered and un-guided missile is called a **ballistic missile**, which is like a projectile.

Chap 4

Work Done by Gravitational Field

"The space around the earth within which it exerts a force of attraction on other bodies." is called **gravitational field.**

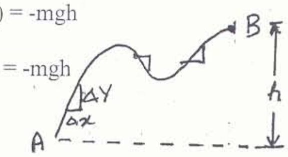
Consider, an object of mass m displaced from point A to B along various paths under gravitational force.



We have, $\vec{W} = \vec{F} \cdot \vec{d} = Fd\cos\theta = mgh\cos\theta$

i) $W_{ADB} = W_{A \to D} + W_{D \to B}$
$= mgd\cos 90^o + mgh\cos 180^o = 0 + mgh(-1) = -mgh$

ii) $W_{ACB} = W_{A \to C} + W_{C \to B}$
$= mgh\cos 180^o + mgd\cos 90^o = mgh(-1) + 0 = -mgh$



iii) Work along the curved path 3

$W_{AB} = \Sigma\, mgd\cos\theta = mg\,(\sum_{i=1}^{n} \Delta x_i + \sum_{i=1}^{n} \Delta y_i)$

$= mg\,(x_1 \cos 90^o + x_2 \cos 90^o + \ldots + y_1 \cos 180^o + y_2 \cos 180^o + \ldots)$

$= mg\,(\sum_{i=1}^{n} \Delta y_i (-1)) = -mgh \qquad [\ \sum_{i=1}^{n} \Delta y_i = h\ ]$

iv) Work along the path B to A

Taking path 1, $W_{BA} = W_{B \to d} + W_{D \to A}$
$= mgh\cos 0^o + mgd\cos 90^o = mgh + 0 = mgh$

(i) to (iii) show that: Work done in the Earth's gravitational field is independent of the path followed.

Adding (iii) & (iv) gives, $W_{A \to B \to A} = W_{AB} + W_{BA} = -mgh + mgh = 0$

So Work done along a closed path in a gravitational field is **zero**

Examples:
Conservative field: i) Gravitational field, ii) Electric field, iii) Magnetic field
Non-conservative field: i) Frictional field, ii) Rough surface, iii) Air resistive field.

Power and Velocity: $P = \frac{\Delta W}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{F} \cdot \Delta\vec{d}}{\Delta t} = \vec{F} \cdot \frac{\Delta\vec{d}}{\Delta t} = \vec{F} \cdot \vec{v}$

Energy: $K.E. = \frac{1}{2}mv^2$; $P.E. = mgh$

Work-Energy Principle: $Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Escape Velocity

Initial velocity, which a projectile must have at earth's surface in order to go out of earth's gravitational field.
& energy required to move a mass from the earth up to an infinite distance is absolute PE.

so $KE_{initial} = PE_{absolute}$

or $\frac{1}{2}mv_{esc}^2 = \frac{GMm}{R}$ or $v_{esc}^2 = \frac{2GM \times R}{R \times R} = \frac{2GMR}{R^2}$

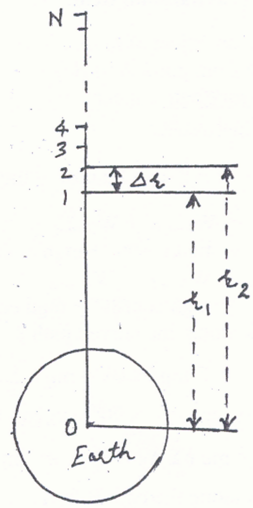
or $v_{esc}^2 = 2R \times \frac{GM}{R^2} = 2Rg$ or $\boxed{v_{esc} = \sqrt{2gR}}$ $\qquad [\ g = \frac{GM}{R^2}\ ]$

Absolute Potential Energy

"Energy required to move a mass from the earth up to an infinite distance".

Consider: mass **m** which moves From point **1** to **N** & dividing 1 to N into Δ**r** steps

We have $\Delta r = r_2 - r_1$ & $r = \frac{r_1 + r_2}{2}$ or (after calculations) $r^2 = r_1 r_2$



$$F = G\frac{Mm}{r^2} \quad \text{or} \quad F = G\frac{Mm}{r_1 r_2}$$

$$W_{1\to 2} = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos 180^\circ = -F\Delta r$$

$$\text{or} \quad W_{1\to 2} = -G\frac{Mm}{r_1 r_2}(r_2 - r_1)$$

$$\text{or} \quad W_{1\to 2} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

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$$W_{N-1\to N} = -GMm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

So the total work done is

$$W_{total} = W_{1\to 2} + \ldots + W_{N-1\to N}$$

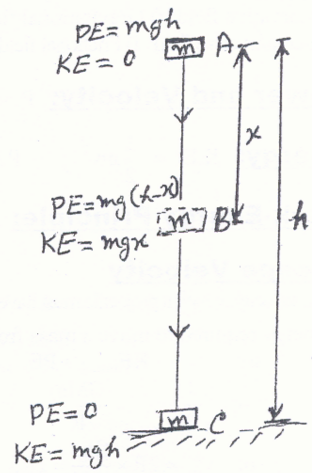
$$\text{or} \quad W_{total} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_N}\right), \quad \frac{1}{r_\infty} = 0$$

$$\text{so} \quad W_{total} = \frac{-GMm}{r_1}, \; r_1 = R \text{, radius of Earth}$$

General expression $\boxed{U = \frac{-GMm}{r}}$ or $\boxed{U_g = -\frac{GMm}{R}}$

Interconversion of PE & KE

Consider, a body of mass m at rest, at a height h above the surface of the Earth.



At point A: Total Energy = T = PE + KE

or $T = mgh + 0 = mgh$

At point B: $[v_f^2 = v_i^2 + 2aS]$; $v_B^2 = 0 + 2gx = 2gx$

Now T = PE + KE

or T = mg(h-x) + ½ m x 2gx

or T = mgh − ~~mgx~~ + ~~mgx~~ = mgh

At point C: $[v_f^2 = v_i^2 + 2aS]$ or $v_C^2 = 0 + 2gh = 2gh$

Now T = PE + KE or T = 0 + ½ m x 2gh = mgh

⇒ Loss of PE = Gain in KE

Assuming a frictional force f is present during the downward motion, then

Total downward force will be (F - f) = (mg -f),

so $(mg - f)\,h\cos 0^\circ = \frac{1}{2} m v^2$

or $mgh - fh = \frac{1}{2} m v^2$

or $mgh = \frac{1}{2} m v^2 + fh$

⇒ Loss of PE = Gain in KE +W. done against friction

Chap 5

Centripetal Force

It is the force needed to bend the normally straight path of the particle into a circular path.

Centripetal Acceleration

The instantaneous acceleration of an object traveling with uniform speed in a circle and is directed towards the center of the circle.

To calculate Centripetal Acceleration

Consider, a particle of mass m,

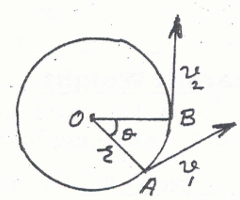
From the figure, $v_1 = v_2 = v$

& $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$ or $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

We have, $\sin\theta = \frac{\Delta v}{v_2}$ or $\theta = \frac{\Delta v}{v}$

or $v\,\Delta\theta = \Delta v$ or $v\frac{\Delta\theta}{\Delta t}\Delta t = \Delta v$

or $\Delta v = v\,\omega\,t$ or $\frac{\Delta v}{\Delta t} = \omega v$

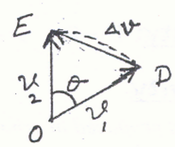


Now we define

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

so $a = \omega v$

or $a = \omega^2 r = \frac{v^2}{r}$



In vector form, $\vec{a} = -\omega^2\vec{r} = -\frac{v^2}{r^2}\vec{r}$ ..... (1)

negative sign indicates that the acceleration is towards the center.

To calculate Centripetal Force

We have, $\vec{F} = m\vec{a}$ ..... (2)

from equations (1) & (2), we get

$$\vec{F} = -m\omega^2\vec{r} = -\frac{mv^2}{r^2}\vec{r} \quad \text{or} \quad F_c = \frac{mv^2}{r}$$

in angular measure, we have $F_c = mr\omega^2$

Moment of Inertia: $I = \sum_{i=1}^{n} m_i r_i^2$

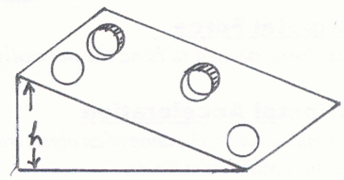
Artificial Satellites

$a_c = g = \frac{v^2}{r} \Rightarrow v = \sqrt{gr} = \sqrt{gR} = 7.9\ \text{kms}^{-1}$, is called **critical velocity.**

$\omega = \frac{\theta}{t} = \frac{2\pi}{T}$ or $T = \frac{2\pi}{\omega} = \frac{2\pi}{v/r} = \frac{2\pi R}{v} = 84$ min, is **Time** for one revolution

The higher the satellite, the slower will be speed and longer time period.
Close orbiting satellites are at a height of 400 km. There are 24 satellites in GPS.

Rotational KE of a Disc & a Hoop



For Disc, PE at the top = total KE at the bottom

$PE_{top} = KE_{trans} + KE_{rot}$

i.e. $mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

or $mgh = \frac{1}{2} mv^2 + \frac{1}{2} (\frac{1}{2} mr^2) (v^2 / r^2)$

or $mgh = \frac{1}{2} m v^2 + \frac{1}{4} m v^2$ or $gh = \frac{3}{4} v^2$

$$\text{or} \quad v^2 = \frac{4gh}{3} \qquad \text{or} \quad v_{disc} = \sqrt{\frac{4gh}{3}}$$

For Hoop, PE at the top = total KE at the bottom

$PE_{top} = KE_{trans} + KE_{rot}$

i.e. $mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} mv + \frac{1}{2} (mr^2) (v^2 / r^2) = \frac{1}{2} m v^2 + \frac{1}{2} m v^2$

or $gh = v^2$ or $v^2 = gh$ or $v_{hoop} = \sqrt{gh}$

so $v_{disc} > v_{hoop}$ when rolls down an inclined plane of height h.

Real & Apparent Weight

For rest position: $T = mg$; For upward motion: $T = mg + ma$

For downward motion: $T = mg - ma$; For freely falling bodies $a = g$, so $T = mg - mg = 0$

An Earth's satellite is freely falling object in space, everything within this freely falling system will appear to be **weightless**.

Orbital Velocity: $F_g = F_c$ or $\frac{Gm_s M}{r^2} = \frac{m_s v^2}{r}$ or $v = \sqrt{\frac{GM}{r}}$

Artificial Gravity

The gravity like effect produced in orbiting space ship to overcome weightlessness.

$$\text{We have,} \quad a = r\omega^2 \quad \text{or} \quad a_c = R\left(\frac{2\pi}{T}\right)^2 = R\left(\frac{2\pi}{1/f}\right)^2 \quad \text{or} \quad a_c = R4\pi^2 f^2$$

$$\text{or} \quad f^2 = \frac{a_c}{R4\pi^2} \qquad \text{or} \quad f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

to increase f so that a_c equals g , and from the above equation, we have

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

Geostationary Orbits

For a satellite, $v = \sqrt{\frac{GM}{r}}$ & for the Earth, $S = vt$ or $v = \frac{S}{t} = \frac{2\pi r}{T}$

$$\Rightarrow \quad \frac{2\pi r}{T} = \sqrt{\frac{GM}{r}} \quad \text{or} \quad \frac{4\pi^2 r^2}{T^2} = \frac{GM}{r} \quad \text{or} \quad r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$$

Communication Satellites

A satellite communication system can be set up by placing several geostationary satellites.

INTELSAT: It is the world's largest commercial satellite communication services provider. Currently it has over 100 members and provides service to over 200 Earth stations in more than 126 countries.

It operates microwave frequencies of 4, 6, 11 & 14 Ghz and has a capacity of 30,000 two way telephone circuits plus three TV channels.

Chap 6

Terminal velocity: *At the end, the extreme, or maximum velocity reached by certain object.*

We have $F_D = 6\pi\eta r v$ & $w = mg$

so the net downward force will be

$$F = w - F_D \quad \text{or} \quad F = mg - 6\pi\eta r v$$

From 2^{nd} Law of motion, $F = ma \Rightarrow ma = mg - 6\pi\eta r v$

$$\text{or} \quad a = \frac{mg - 6\pi\eta r v}{m} \quad \text{or} \quad a = g - \frac{6\pi\eta r v}{m}$$

$$0 = g - \frac{6\pi\eta r v_T}{m} \quad \text{or} \quad 6\pi\eta r v_T = mg$$

At terminal velocity, $v = v_T$ & $a = 0$

$$\text{or} \quad v_T = mg / 6\pi\eta r$$

$$\text{or} \quad v_T = \frac{(\frac{4}{3}\pi r^3)\rho g}{6\pi\eta r}$$

$$\rho = \frac{\text{mass}}{\text{vol.}} = \frac{m}{\frac{4}{3}\pi r^3}$$

$$\text{or} \quad m = (\tfrac{4}{3}\pi r^3)\rho$$

$$\text{or} \quad \boxed{v_T = \frac{2\rho g r^2}{9\eta}} \Rightarrow \boxed{v_T \propto r^2}$$

Short Cut: $(F - f) = ma$ *or* $mg - 6\pi\eta r v = ma$ *or* $g - \frac{6\pi\eta r v_T}{m} = 0$

$$\text{or} \quad v_T = \frac{mg}{6\pi\eta r} = \frac{(\frac{4}{3}\pi r^3)\rho g}{6\pi\eta r} \quad [m = V\rho] \quad \text{or} \quad v_T = \frac{2\rho g r^2}{9\eta}$$

Equation of Continuity: $A_1 v_1 = A_2 v_2$

The fluid is streamline and incompressible.

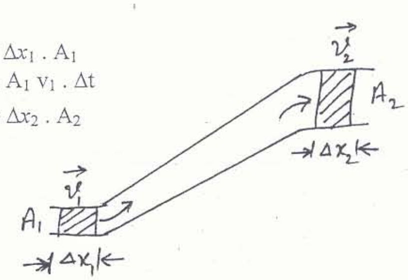
At left side: $v_1, \Delta x_1, A_1$, So volume $= V_1 = \Delta x_1 \cdot A_1$

& $\Delta m_1 = \rho_1 V_1 = \rho_1 \Delta x_1 \cdot A_1$ or $\Delta m_1 = \rho_1 A_1 v_1 \cdot \Delta t$

At right side: $v_2, \Delta x_2, A_2$ So volume $= V_2 = \Delta x_2 \cdot A_2$

$\Delta m_2 = \rho_2 V_2 = \rho_2 \Delta x_2 \cdot A_2$

or $\Delta m_2 = \rho_2 A_2 v_2 \cdot \Delta t$



As the streamline flow is incompressible,

so $\Delta m_1 = \Delta m_2$

$\Rightarrow \rho_1 A_1 v_1 \cdot \Delta t = \rho_2 A_2 v_2 \cdot \Delta t$

since density is constant, i.e., $\rho_1 = \rho_2 = \rho$

so $\rho A_1 v_1 = \rho A_2 v_2$

or $A_1 v_1 = A_2 v_2$, which is Equation of Continuity.

Short Cut: $\rho = \frac{m}{V}$ or

$m = \rho V = \rho \times \Delta x \cdot A = \rho A v \Delta t$ $[S = vt]$

as $\Delta m_1 = \Delta m_2$

Bernoulli's Equation: $P + ½ \rho v + \rho gh = constant$

Assuming the fluid is , i) incompressible, ii)Non-viscous, iii) streamline flow

Let a liquid of mass (Δm), flowing through a pipe during time (t),

At left side: $P_1, v_1, \Delta x_1, A_1, h_1$ & At right side: $P_2, v_2, \Delta x_2, A_2, h_2$

So $W = F \times \Delta x = PA\ \Delta x$ [$P = F / A$ or $F = P A$]

Also $S = \Delta x = v t$ & $\rho = m / V$ or $V = m / \rho$

so $A . \Delta x = A v t = V = m / \rho$ [$V = A / \ell$]

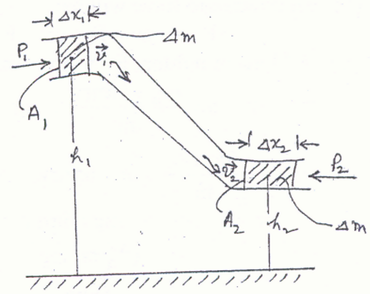
for the same mass flowing during time t, through both ends, the volume will be

$A_1 v_1 t = A_2 v_2 t = A v t$

$\Rightarrow W = P A v t$ or $W = P m / \rho$

Now we have , $KE = ½ m v^2$ & $PE = m g h$

Same Δm is flowing from upper to lower end, so from Law of conservation of energy



Net Work done = change in KE + change in PE

or $W_{upper\ end} + W_{lower\ end} = \{KE_{upper} - KE_{lower}\} + \{PE_{upper} - PE_{lower}\}$

$P_1 m / \rho + \{(-P_2) m / \rho\} = ½ m v_2^2 - ½ m v_1^2 + m g h_2 - m g h_1$

or $m / \rho (P_1 - P_2) = m (½ v_2^2 - ½ v_1^2 + g h_2 - g h_1)$

or $P_1 - P_2 = \rho (½ v_2^2 - ½ v_1^2 + g h_2 - g h_1)$

or $P_1 - P_2 = ½ \rho v_2^2 - ½ \rho v_1^2 + \rho g h_2 - \rho g h_1$

or $P_1 + ½ \rho v_1^2 + \rho g h_1 = P_2 + ½ \rho v_2^2 + \rho g h_2$

or $P + ½ \rho v^2 + \rho g h =$ constant, which is Bernoulli's Equation.

Short Cut: Net $W = \Delta KE + \Delta PE$ or $W_U + W_L = KE_U - KE_L + PE_U - PE_L$

$$W = Fd = PAd = PV = P\frac{m}{\rho} \qquad [P = F/A \ \& \ \rho = m/V]$$

$$\text{or } P_1\frac{m}{\rho} + (-P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1 \quad \text{or} \quad P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gh_2 - \rho gh_1$$

$$\text{or } P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 \quad \text{or} \quad P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

Applications: Torricelli's Theorem

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 \qquad [v_2 \gg v_1, \text{ so ignoring top velocity } v_1]$$

$$\text{or } P_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 \quad \text{or} \quad \Rightarrow v_2 = \sqrt{2g(h_1 - h_2)} \qquad [P_1 = P_2]$$

Venturi Relation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 \qquad [A_2 \ll A_1 \Rightarrow v_1 \ll v_2 \ \& \ h_1 = h_2]$$

or $P_1 - P_2 = \frac{1}{2}\rho v_2^2$, called Venturi relation.

Relation between Speed & Pressure

$$P_A + \frac{1}{2}\rho v_A^2 + \rho gh_A = P_B + \frac{1}{2}\rho v_B^2 + \rho gh_B \qquad [\rho gh_A = \rho gh_B]$$

$\Rightarrow P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$, Shows *where the speed is high, the pressure will be low.*

Chap 7

Consider a mass m attached to one end of an elastic spring which can, move freely on a frictionless horizontal surface.

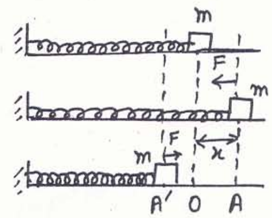
The restoring force = F = -k x

& we have F = m a

$\Rightarrow$ -k x = m a or $a = -\frac{k}{m}x$

or $a = -(\text{const.})\,x$ or $a \propto -x$

which is characteristics of **SHM**

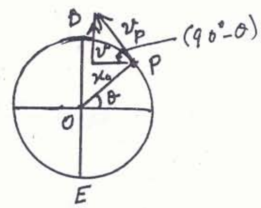


SHM & Uniform Circular Motion

Consider, a point P with angular speed ω & radius x_o

so $v = x_o\,\omega$ $[v = r\,\omega]$

We have $\omega = \frac{\theta}{t}$ or $t = \frac{\theta}{\omega}$, $\theta = 2\pi$ rad so $T = \frac{2\pi}{\omega}$

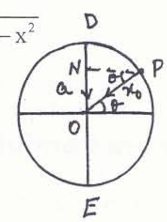


From the figure $x = x_o \sin\omega t$

& $v = v_P \sin(90^0 - \theta) = v_P \cos\theta = x_o\omega\cos\theta$ $[v_P = x_o\omega]$

or $v = x_o\omega\sqrt{1-\sin^2\theta} = x_o\omega\sqrt{1-\frac{x^2}{x_o^2}} = x_o\omega\sqrt{\frac{x_o^2-x^2}{x_o^2}}$

or $v = x_o\omega\sqrt{\frac{1}{x_o^2}(x_o^2-x^2)} = \frac{x_o\omega}{x_o}\sqrt{x_o^2-x^2} = \omega\sqrt{x_o^2-x^2} \Rightarrow v = \omega\sqrt{x_o^2-x^2}$



Now $a_c = \frac{v_P^2}{x_o} = \frac{x_o^2\omega^2}{x_o} = x_o\omega^2$ $[a = \frac{v^2}{r}$ & $v_P = x_o\omega]$

its component along the diameter DOE is, $a = x_o\omega^2\sin\theta$

since it is directed towards center and $x = x_o\sin\theta$, so $a = -\omega^2 x$

Thus the point N has acceleration proportional to displacement and directed towards the center, which is the characteristic of SHM. So the projection of P executes SHM. We can define SHM as "the projection of uniform circular motion upon any diameter of a circle".

A Horizontal Mass Spring System:

Consider, the vibrating mass attached to a spring whose **acceleration** at any instant is given

by $a = -\frac{k}{m}x$ also we have $a = -\omega^2 x$ $\Rightarrow$ $\omega = \sqrt{\frac{k}{m}}$

so $T = \frac{2\pi}{\omega}$ or $T = \frac{2\pi}{\sqrt{k/m}}$ or $T = 2\pi\sqrt{\frac{m}{k}}$

& we have $x = x_o \sin\omega t$ or $x = x_o \sin\sqrt{\frac{k}{m}}\,t$

& from $v = \omega\sqrt{x_o^2-x^2}$ or $v = \sqrt{\frac{k}{m}}\sqrt{x_o^2-x^2} = \sqrt{\frac{k}{m}(x_o^2-x^2)} = \sqrt{x_o^2\left\{\frac{k}{m}\left(\frac{x_o^2-x^2}{x_o^2}\right)\right\}}$

or $v = x_o\sqrt{\frac{k}{m}\left(1-\frac{x^2}{x_o^2}\right)}$ or $v_o = x_o\sqrt{\frac{k}{m}}$ $\Rightarrow v = v_o\sqrt{1-\frac{x^2}{x_o^2}}$

Energy Conservation in SHM

We have P.E. = W. done $= F_{av} \cdot x = \frac{1}{2}kx \cdot x = \frac{1}{2}kx^2$

or $P.E. = \frac{1}{2}kx^2 \Rightarrow P.E._{max} = \frac{1}{2}kx_o^2$

& $P.E._{min} = \frac{1}{2}k(0)^2$ or $P.E._{min} = 0$

And $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(x_o\sqrt{\frac{k}{m}\left(1-\frac{x^2}{x_o^2}\right)}\right)^2 = \frac{1}{2}mx_o^2\frac{k}{m}\left(1-\frac{x^2}{x_o^2}\right)$

or $K.E. = \frac{1}{2}kx_o^2\left(1-\frac{x^2}{x_o^2}\right) \Rightarrow K.E._{max} = \frac{1}{2}kx_o^2$

& $K.E._{min} = \frac{1}{2}kx_o^2\left(1-\frac{x_o^2}{x_o^2}\right) = \frac{1}{2}kx_o^2(1-1) = \frac{1}{2}kx_o^2 \times 0 \Rightarrow K.E._{min} = 0$

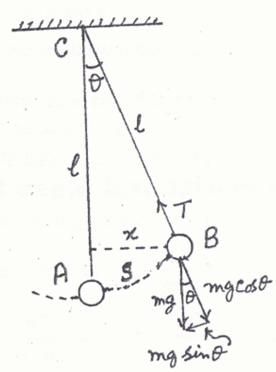
Now $E_{total} = P.E. + K.E.$

$$= \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2\left(1-\frac{x^2}{x_o^2}\right) = \frac{1}{2}kx^2 + \left(\frac{1}{2}kx_o^2 - \frac{1}{2}kx^2\right)$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2 - \frac{1}{2}kx^2 \quad \text{or} \quad E_{total} = \frac{1}{2}kx_o^2$$

⇒ The energy oscillates back and forth between kinetic energy and potential energy but total energy of the mass remains constant everywhere.

Simple Pendulum



Let the bob is at position B during its vibratory motion.
Two forces are acting, Weight mg & Tension T
mg is resolved into two components, $mg\cos\theta$ & $mg\sin\theta$

$T = mg\cos\theta$

Component of mg perpendicular to the string = $mg\sin\theta$

We have $F = ma$

$\Rightarrow ma = -mg\sin\theta$ or $a = -g\sin\theta$

or $a = -g\theta = -g\frac{x}{l} = -\frac{g}{l}x \Rightarrow a \propto -x$

Now we have $a = -\omega^2 x$, $\Rightarrow \omega^2 = \frac{g}{l}$ or $\omega = \sqrt{\frac{g}{l}}$

We have time period from general expression of SHM

From $T = \frac{2\pi}{\omega}$ gives $T = \frac{2\pi}{\sqrt{g/l}} \Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$

Free Oscillations: When a body oscillates without the interference of an external force.

Forced Oscillations: When a freely oscillating system is subjected to an external force.

Damped Oscillator: The oscillator in which the amplitude decreases steadily with time.

Damping is the process whereby energy is dissipated from the oscillatory system.

A heavily damped system has a fairly flat resonance curve.

Chap 8

Newton's formula for the velocity of sound in air: $v = \sqrt{\frac{E}{\rho}}$

To prove: $E = P$, We have $P_1 V_1 = P_2 V_2$ or $PV = (P + \Delta P)(V - \Delta V)$

or $PV = PV - P\Delta V + V\Delta P - \Delta P \Delta V$ [Neglecting $\Delta P \Delta V$]

we get $P\Delta V = V\Delta P$ or $P = \Delta P \frac{V}{\Delta V} = \frac{\Delta P}{\Delta V / V} = \frac{\text{stress}}{\text{strain}} = E$

or $P = E \Rightarrow v = \sqrt{\frac{P}{\rho}}$

Laplace's Correction: Taking $PV^\gamma = \text{Constant}$

or $PV^\gamma = (P + \Delta P)(V - \Delta V)^\gamma$ or $PV^\gamma = (P + \Delta P)\left\{\frac{(V - \Delta V)V}{V}\right\}^\gamma$

or $PV^\gamma = (P + \Delta P)\left\{1 - \frac{\Delta V}{V}\right\}^\gamma V^\gamma$ or $P = (P + \Delta P)\left\{1 - \frac{\Delta V}{V}\right\}^\gamma$

or $P = (P + \Delta P)\left(1 - \gamma\frac{\Delta V}{V} - \frac{\gamma(\gamma - 1)}{1 \cdot 2}\frac{(\Delta V)^2}{V^2} - \dots\dots\dots\right)$ or $P = (P + \Delta P)\left(1 - \gamma\frac{\Delta V}{V}\right)$

or $P = P - P\gamma\frac{\Delta V}{V} + \Delta P - \Delta P\gamma\frac{\Delta V}{V}$ or $\Delta P = P\gamma\frac{\Delta V}{V}$

or $\gamma P = \frac{\Delta P}{\Delta V / V} = \frac{\text{stress}}{\text{strain}} = E \Rightarrow v = \sqrt{\frac{\gamma P}{\rho}}$.

Effect of Pressure: As density $\propto$ pressure, so v is not affected by change of P.

Effect of Density: For the same temperature, pressure & $\gamma \Rightarrow v \propto \frac{1}{\sqrt{\rho}}$

Effect of Temperature: We have $v = \sqrt{\frac{\gamma P}{\rho}}$

$$\Rightarrow v_o = \sqrt{\frac{\gamma P}{\rho_o}} \quad \& \quad v_t = \sqrt{\frac{\gamma P}{\rho_t}} \quad \text{or} \quad \frac{v_t}{v_o} = \frac{\sqrt{\gamma P / \rho_t}}{\sqrt{\gamma P / \rho_o}} = \sqrt{\frac{\rho_o}{\rho_t}}$$

Now we have $V_t = V_o(1 + \beta t) \Rightarrow V_t = V_o\left(1 + \frac{t}{273}\right)$

$$\Rightarrow \frac{v_t}{v_o} = \sqrt{1 + \frac{t}{273}} \quad \text{or} \quad \frac{v_t}{v_o} = \sqrt{\frac{273 + t}{273}} = \sqrt{\frac{T}{T_o}} \Rightarrow \boxed{v = \sqrt{T}}$$

i.e. speed of sound varies directly as the square root of absolute temperature.

$$\Rightarrow v_t = v_o + \frac{332 \times t}{546} \quad \text{or} \quad \boxed{v_t = v_o + 0.61 \times t}$$

Interference

Superposition of two waves having the same frequency and traveling in the same direction results interference.

Condition for constructive interference: $\Delta S = n\lambda$, $n = 0, \pm1, \pm2, \pm3, \pm4, \ldots$

Condition for destructive interference: $\Delta S = (2n+1)\frac{\lambda}{2}$, $n = 0, \pm1, \pm2, \pm3, \pm4 \ldots$

Beats

The condition whereby two sound waves form an outburst of sound followed by an interval of comparative silence.

Number of beats per second is equal to difference between the frequencies of tuning forks.

Applications: Used in finding the unknown frequencies & in tuning the musical instruments.

Stationary Waves in a Stretched String

$\lambda_n = \frac{2}{n}\ell$ and $f_n = nf_1$ & $f_1 = \frac{1}{2\ell}\sqrt{\frac{F}{m}}$

Stationary Waves in Air Columns

A pipe open at both ends $\lambda_n = \frac{2}{n}\ell$ & $f_n = nf_1$ where $n = 1, 2, 3, 4, 5, \ldots\ldots\ldots$

A pipe closed at one end & open at other: $\lambda_n = \frac{4\ell}{n}$ & $f_n = n\frac{v}{4\ell}$ $n = 1, 3, 5, \ldots\ldots..$

⇒ The pipe closed at one end have only odd harmonics & the pipe, which is open at both ends, is rich in harmonics.

DOPPLER EFFECT

The change in the frequency of the waves caused by the relative motion of either the source of waves or the observer.

1. **Observer is moving towards the stationary source:** $f_A = f\left(\frac{v+u_o}{v}\right)$

   As $f_A > f$, therefore the frequency heard / observed by the observer will increase.

2. **Observer is receding from the stationary source:** $f_B = f\left(\frac{v-u_o}{v}\right)$

   As $f_B < f$, therefore the frequency heard / observed by the observer will be reduced.

3. **Source is moving towards the stationary observer:** $f_C = \left(\frac{v}{v-u_S}\right)f$

   As $f_C > f$, therefore the frequency heard / observed by the observer will increase.

4. **Source is moving away from the stationary observer:** $f_D = \left(\frac{v}{v+u_S}\right)f$

   As $f_D < f$, therefore the frequency heard / observed by the observer will be decreased.

<u>Applications</u>:

1)Ultrasonic waves from a bat, 2) Reflection of radar waves,

3) Radar speed trap:

Microwaves are emitted from a transmitter in short bursts. Each burst is reflected off by any car in path. By measuring Doppler shift, speed of car is calculated by computer.

4) Applied to light:

Velocities of earth satellites are determined from the Doppler shift.

Chap 9

Young's Double Slit Experiment

Young's double-slit experiment gives the experimental evidence for Huygen's wave theory of light.

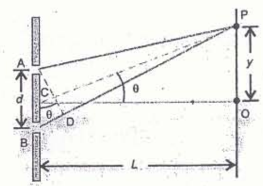
To derive The equation for maxima and minima,

For P to be bright fringe, $\Delta S = m\lambda$, or $d \sin\theta = m\lambda$, $m = 0, 1, 2, \ldots$

For dark fringes, $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ $m = 0,1,2,3,\ldots..$

Now from the figure, $\tan\theta = \frac{y}{L}$ or $y = L\tan\theta$ or $y = L\sin\theta$

$$\Rightarrow \quad \sin\theta = m\frac{\lambda}{d}$$



so $y = m\frac{\lambda L}{d}$ or $\lambda = \frac{yd}{mL}$

$\Rightarrow$ Position of m^{th} bright fringe = $y_m = m\frac{\lambda L}{d}$

& (m + 1)th bright fringe: $y_{m+1} = (m+1)\frac{\lambda L}{d}$

so fringe width = $\Delta y = y_{m+1} - y_m = (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d}$ or $\Delta y = \frac{\lambda L}{d}$

Interference in thin films

The transparent medium whose thickness is comparable with the wavelength of light is called thin film.

The monochromatic light will look bright or dark depending upon their path difference to make constructive or destructive interference.

Part I of this beam is reflected from the upper surface and the remaining portion refracted into the film, after reflection from the second face, it emerges out as part II. As the film is thin, so the separation between part I and II is very small and they superimpose on each other. The portions I and II, being the parts of same beam, will have phase coherence. So the effect of their interference can be detected. This results in the formation of circular rings.

Path difference depends upon: 1. Thickness of the film. 2. Angle of incidence.
3. Nature depending upon index of refraction, n.

Examples:

1) Coloured fringes formed in soap bubbles. 2) Formation of colours in thin layer of oil on water.
3) Formation of colours on metal sheets during welding.

Newton's Rings

Coloured rings produced by the interference of light waves.

A plano convex lens when placed in contact with a plane glass surface. And illuminated from above by a parallel beam of mono-chromatic light, a series of concentric rings are observed. They are formed due to the interference between rays reflected by the top and bottom surfaces of air gap between the convex lens and the plane glass.
The air gap, equivalent to a thin film, increases in width from the central contact point out to the edges, corresponds to constructive and destructive interference and results in series of bright and dark rings.

Michelson's Interferometer

Michelson's interferometer is an instrument that can be used to measure distance with extremely high precision.
Device includes one half silvered mirror and two plane mirrors, using interference of light waves to measure very small distances.
It splits a light beam into parts and then recombines them to form an interference pattern. It is used for accurate measurement of wavelength.

A fringe is shifted, each time when one of the mirror is displaced through $\lambda/2$. Hence counting the number m of the fringes, which are shifted by the displacement L of the mirror, we can write the equation, $L = m\frac{\lambda}{2}$

This interference is used to make very accurate measurements.

Diffraction of Light

The property of bending of light around obstacles and spreading of light waves into the geometrical shadow of an obstacle.

Diffraction Grating

A diffraction grating is a glass plate having a large number of close parallel equidistant slits mechanically ruled on it. To make constructive interference. We have

$d \sin\theta = n\lambda$, $n = 0, \pm1, \pm2, \pm3, \ldots$ where n is number of order of image.

Diffraction of X-Rays by Crystals

***X-rays** is a type of electromagnetic radiation with wavelength less than 10^{-10} m.*
The regular array of atoms in a crystal forms a natural grating with spacing $\sim 10^{-10}$ m.
In the atomic planes of constant inter-planar spacing, when an X-ray beam is incident at an angle θ on one of the planes, the beam will be reflected from both upper and lower planes.
The path difference will be, $2d\sin\theta = n\lambda$ where n is the order of reflection.

The above equation is known as Bragg equation.
It can be used to determine inter-planar spacing between similar parallel planes of a crystal if known wavelength of X-rays is used.
X-ray diffraction has been useful determining the structure of biologically important molecules such as hemoglobin and double helix structure of DNA. It is also used in the field of X-ray crystallography in which structure of crystals is investigated.

Polarization

The limiting of the vibrations of light, usually to vibrations in one plane.

The phenomenon of interference and diffraction proves the wave nature of light, but polarization shows that light moves as transverse waves.

When two polaroids placed with their crystal axes parallel, a beam of light falls on them is transmitted. If one of them is rotated, the intensity of the transmitted light decreases and finally cut off when the axes of two crystals become perpendiculars to each other. On further rotation the light reappears. This transmitted light is called plane polarized.
Polarization depends upon: 1) Selective absorption of light. 2) Reflection of light.
3) Refraction of light. 4) Scattering of light.

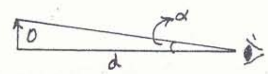
Applications: 1) Polaroid filters, 2) Curtain-less window
3) Concentration of sugar in blood or urine is determined through polarized light.
4) Head-light glare can be controlled by polarizing headlights and light polarizing viewer.
5) In Photography Polarizing discs are used in front of camera lens to enhance the effect of sky.

Chap 10

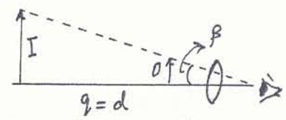
An ordinary convex lens held close to the eye is called **simple microscope**.

Magnifying Power: $M = \dfrac{\beta}{\alpha}$, $\tan\alpha = \alpha = \dfrac{O}{d}$ & $\tan\beta = \beta = \dfrac{I}{q} = \dfrac{I}{d}$ $\Rightarrow$ $M = \dfrac{I/d}{O/d} = \dfrac{I}{O}$

From the figure, $\dfrac{I}{O} = \dfrac{d}{p}$ $\Rightarrow$ $M = \dfrac{d}{p}$



Now from the lens formula, $\dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{-q} = \dfrac{1}{p} - \dfrac{1}{d}$

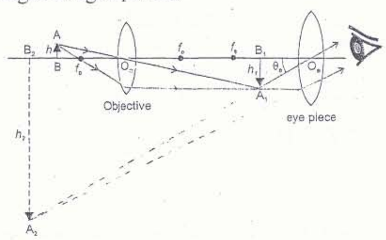


or $\dfrac{d}{f} = \dfrac{d}{p} - \dfrac{d}{d} = \dfrac{d}{p} - 1$ or $\dfrac{d}{p} = 1 + \dfrac{d}{f}$ $\Rightarrow M = 1 + \dfrac{d}{f}$

Compound Microscope:

It is a device used to produce a very large magnification of very small objects. It consists of an objective and an eyepiece. Object AB of height h forms a real, inverted & enlarged image h_1 of the object placed within focal length of eyepiece.

Magnifying Power: $M = \dfrac{\beta}{\alpha} = \dfrac{\tan\beta}{\tan\alpha} = \dfrac{\tan\theta_e}{\tan\theta}$



or $M = \dfrac{h_2/d}{h/d} = \dfrac{h_2}{h}$

or $M = \dfrac{h_2}{h} \times \dfrac{h_1}{h_1} = \dfrac{h_1}{h} \times \dfrac{h_2}{h_1} = M_1 M_2$

Now in the figure, $\dfrac{A_1B_1}{AB} = \dfrac{B_1O}{BO}$ or $\dfrac{A_1B_1}{AB} = \dfrac{h_1}{h} = M_1 = \dfrac{q}{p}$

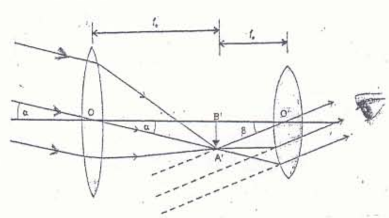
& $M_2 = \dfrac{h_2}{h_1} = 1 + \dfrac{d}{f_e}$ $\Rightarrow$ $M = \dfrac{q}{p}\left(1 + \dfrac{d}{f_e}\right)$, $f_o < f_e$

Astronomical Telescope:

It is a telescope used to see heavenly bodies; it consists of two convex lenses, one for objective and the other as an eyepiece.
Objective lens is of large f with large aperture. The eyepiece has short f and small aperture. Objective form a real, inverted and diminished image at its focus of a distant object, in front of eyepiece. The distance between the eyepiece and this image is adjusted within the focal length so that a magnified and virtual image is formed at the least distance of distinct vision. If the image $A'B'$ is made at focus of eyepiece then the final image is formed at infinity. It is called the telescope is focused for infinity.
Then, Length of the telescope = $f_o + f_e$

Magnifying power: $M = \dfrac{\beta}{\alpha}$, $\alpha = \tan\alpha = \dfrac{A'B'}{B'O}$

& $\beta = \tan\beta = \dfrac{A'B'}{B'O'}$



$\Rightarrow$ $M = \dfrac{A'B'/f_o}{A'B'/f_e} = \dfrac{A'B'}{f_e} \times \dfrac{f_o}{A'B'}$

or $M = \dfrac{f_o}{f_e}$, $f_e < f_o$

Optical Fibres

An optical fibre consists of a single flexible rod of high refractive index, less than 1mm in diameter, having polished surfaces coated with transparent material of lower refractive index.

Fibre Optic Principles

Propagation of light in an optical fibre should be totally confined within the fibre. This can be done by: **1) Total Internal Reflection & 2) Continuous Refraction**

Optical fibres consist of i) glass core, ii) glass cladding, & iii) jacket

1. **Single (or mono) Mode Index Fibre**

   An optical fibre having a very thin core of about 5 μm diameter and has a relatively larger cladding of glass or plastic.

   Core diameter: 5 μm; **Source:** laser light; **Capacity:**14 TV channels or14000 phone calls.
2. **Multimode Step Index Fibre**

   An optical fibre having a core of relatively larger diameter such as 50 μm is used. The fibre core has a constant refractive index such as 1.52.

   Core diameter: 50 μm; **Light source:** White light; **Useful:** For short distance only
3. **Multimode Graded Index Fibre**

   An optical fibre in which the central core has high refractive index which gradually decreases towards its periphery.

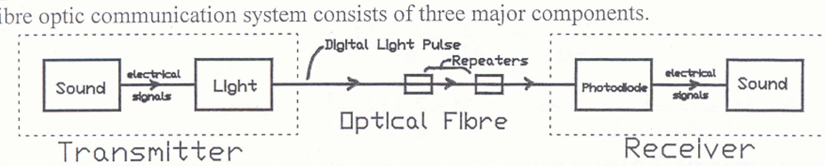
   Core diameter: 50 to 1000 μm; **Source:** White light; **Useful:** For long distance.

Applications

1)Transmission to inaccessible place, 2) Image transmission in facsimile systems.
3) used in medical instruments & 4) In telecommunications

Signal Transmission & Conversion to Sound:

A fibre optic communication system consists of three major components.



Losses of Power:

Factors: i) Scattering, ii) Absorption & iii) Dispersion

Results: Faulty & distorted signals are received.

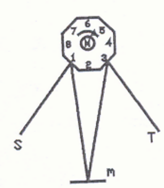
Remedies: Use graded index fibre instead step index fibre.

Efficiency: Time difference is reduced by 1 ns per km instead of 33 ns per km.

Spectrometer: *Optical instrument used for the study of spectra from different sources of light. It consists of* ***collimator, turntable*** *and* ***telescope.***

Speed of Light

Michelson's experiment: The speed of light was determined by measuring time it took to cover a round trip between two mountains. The eight-sided mirror M can be rotated clockwise. The source S becomes visible when time taken by light for round trip between M and m is equal to moving the mirror M from face 2 to face 1.



$$t_1 = \frac{2d}{c} \;\&\; t_2 = \frac{1}{2\pi f} \times \frac{2\pi}{8} = \frac{1}{8f} \;\Rightarrow\; \frac{2d}{c} = \frac{1}{8f} \quad \text{or } c = 16\,f\,d$$

Presently accepted value: $c = 2.99792458 \times 10^8 \text{ms}^{-1}$ rounded to $3.00 \times 10^8 \text{ms}^{-1}$

Chap 11

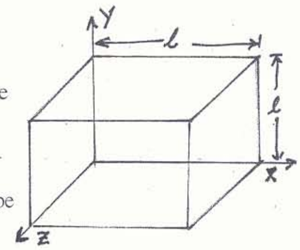
Kinetic Theory of Gases

Gas consists of small molecules, moving in all directions with different velocities, have elastic collisions, moving independendly & molecular radii < mean free path.

Pressure of Gas

Consider a cubical container with side ℓ, Area A & Volume $V = \ell \times \ell^2 = \ell^3$
Having velocity v, time t, & distance traveled = v t = 2ℓ, so t = $2\ell / v_{1x}$
& $M_i = m v_{1x}$ & $M_f = - m v_{1x}$ so $\Delta M = - m v_{1x} - (m v_{1x}) = -2 m v_{1x}$

& rate of change of momentum $= \dfrac{-2mv_{1x}}{2\ell/mv_{1x}} = \dfrac{-mv_{1x} \times v_{1x}}{\ell} = -\dfrac{mv_{1x}^2}{\ell}$



Now $F_{1x} = m a = \dfrac{m(v_f - v_i)}{t}$ = Rate of change of momentum

$\Rightarrow F_{1x} = \dfrac{-mv_{1x}^2}{\ell}$, it is the force exerted by the wall on a molecule

So force exerted on the wall $= -F_{1x} = \dfrac{-mv_{1x}^2}{\ell}$ or $F = \dfrac{mv_{1x}^2}{\ell}$

& total force exerted on right wall by all the molecules, F_x, will be

$$F_x = \frac{mv_{1x}^2}{\ell} + \frac{mv_{2x}^2}{\ell} + \dots\dots\dots + \frac{mv_{Nx}^2}{\ell} \quad \text{or} \quad F_x = \frac{m}{\ell}\Sigma v_{ix}^2$$

So $P_x = \dfrac{F}{A} = \dfrac{m/\ell \, \Sigma v_{1x}^2}{A(=\ell^2)} = \dfrac{m}{\ell^3}\Sigma v_{ix}^2 = \dfrac{\rho}{N}\Sigma v_{ix}^2$

or $P_x = \dfrac{\rho}{N} N < v_x^2 > \; = \rho < v_x^2 >$

or $\rho = \dfrac{m}{\ell^3}$
for n molecules
$\rho = \dfrac{mN}{\ell^3}$ or $\dfrac{m}{\ell^3} = \dfrac{\rho}{N}$

taking $<v^2_x> = <v^2_y> = <v^2_z> = 1/3 <v^2>$ so $P_x = \dfrac{\rho}{3} < v^2 >$

$\rho = \dfrac{mN}{\ell^3} = \dfrac{mN}{V}$

In general form, $P = \dfrac{1}{3}\rho < v^2 > \; = \dfrac{mN}{3V} < v^2 >$

or $P = \dfrac{2}{3}\dfrac{N}{V} < \dfrac{1}{2}mv^2 > \; = \dfrac{2}{3} N_o < \dfrac{1}{2}mv^2 >$ or P = const.x <K.E.> or $P \propto$ <K.E.>

Interpretation of Temperature

We have $PV = nRT$ or $P = nRT/V$ $\Rightarrow$ $\dfrac{nRT}{V} = \dfrac{2}{3}\dfrac{N}{V} < \dfrac{1}{2}mv^2 >$

or $T = \left(\dfrac{2}{3}\dfrac{N}{nR}\right) < \dfrac{1}{2}mv^2 >$ or $T = \text{const.} \times < \dfrac{1}{2}mv^2 >$ or $T \propto < \dfrac{1}{2}mv^2 >$

Derivation of Gas Laws: We have, $P = \dfrac{2}{3}\dfrac{N}{V} < \dfrac{1}{2}mv^2 >$ ..... (1)

If average KE = $<1/2\ mv^2>$ is constant, then

$P = \text{Const.} \times \dfrac{2}{3}\dfrac{N}{V}$ or $PV = \text{Const.} \times \dfrac{2}{3}N = \text{Constant}$ or $P \propto \dfrac{1}{V}$ which is **Boyles's Law.**

Now from eq. (1), $V = \dfrac{2}{3}\dfrac{N}{P} < \dfrac{1}{2}mv^2 >$

If pressure P is constant then, V = constant x $<1/2\ mv^2>$ or $V \propto (KE)_{av}$
As $(KE)_{av}$ is measure of Temperature T, so $V \propto T$ which is **Charles' Law.**

Internal energy (ΔU)

The sum of all forms of molecular energies (kinetic and potential) of a substance.

Internal energy retained is in the form of, KE_{trans}, KE_{vib} & KE_{rot}

Generally internal energy of an ideal gas system is its translational K.E.

Internal energy depends only upon initial and final states, Internal energy is a function of state

Work & Heat

Heat Q ADDED or heat IN is **+ve;** Heat Q LEAVES or OUT is **-ve**

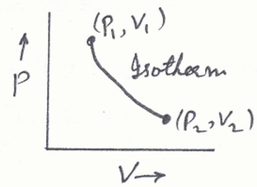
Work is done BY the system is **+ve;** Work is done ON the system is **-ve**

First Law of Thermodynamics

The heat energy supplied to a system is equal to the increase in the internal energy of the system from an initial value U_i to the final value U_f plus the work done by the system on its surroundings. Mathematically $Q = \Delta U + W$

The above equation defines the change in the internal energy of a system. It is equal to the energy flowing in as heat energy minus the energy flowing out as work.

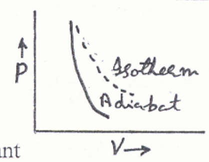
The first law of thermodynamics indicates that there exists a useful state variable of every thermodynamic system called the internal energy.



Isothermal Process: *The process in which the temperature of the system remains constant.*

Gas expand or compress isothermally, so Boyle's Law is fulfilled: $P_1 V_1 = P_2 V_2$ $[\Delta U = 0]$

Applying the equation, $Q = \Delta U + W$ or $Q = W$



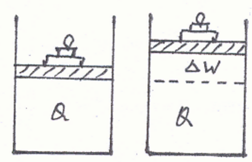
Adiabatic Process: *The process in which no heat enters or leaves the system.*

Applying the equation, $Q = \Delta U + W$ $[Q = 0]$

$\Rightarrow 0 = \Delta U + W$ or $W = -\Delta U$

Also in adiabatic changes we have, $pV^{\gamma} = \text{constant}$

Molar Specific Heats of a Gas



At constant volume: $Q = nC_V\Delta T = \begin{Bmatrix}\text{Heat energy used in raising} \\ \text{the temperature through } \Delta T\end{Bmatrix}$

& $\begin{Bmatrix}\text{Heat energy used in} \\ \text{doing the external work}\end{Bmatrix} = \Delta W = PAd = P\Delta V = nR\Delta T$

At constant pressure: $Q = nC_P\Delta T = \begin{Bmatrix}\text{Heat energy used in raising} \\ \text{the temperature through } \Delta T\end{Bmatrix} + \begin{Bmatrix}\text{Heat energy used in} \\ \text{doing the external work}\end{Bmatrix}$

$\Rightarrow nC_p\,\Delta T = nC_V\,\Delta T + nR\Delta T$ or $C_p = C_V + R$ or $C_p - C_V = R$ $\Rightarrow C_p > C_V$

A Heat Engine must have**:** i) A **source**, which can supply heat, ii) **Sink** to reject heat. & iii) Working **substance.**

Second Law of Thermodynamics

Lord Kelvin's Statement

It is impossible to devise a process which may convert heat, extracted from a single reservoir, entirely into work without leaving any change in the working system.

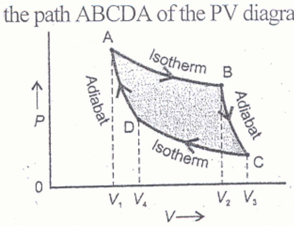
As a consequence of this law, two bodies at different temperatures are essential for the conversion of heat into work. A single heat reservoir, no matter how much energy it contains, cannot be made to perform any work.

Carnot Engine

Sadi Carnot described an ideal engine using only isothermal and adiabatic processes. It is most efficient reversible ideal engine. It consists of four steps. Carnot cycle with ideal gas is shown on PV diagram.

1. Gas is allowed to expand isothermally at temp. T_1, absorbing heat Q_1 from hot reservoir.
2. The gas is then allowed to expand adiabatically until its temperature drops to T_2.
3. The gas is compressed isothermally at temp. T_2 , rejecting heat Q_2 to cold reservoir.
4. Finally the gas is compressed adiabatically to restore its initial state at temperature T_1.

The net work done during one cycle equals to area enclosed by the path ABCDA of the PV diagram.



Applying 1[st] law of thermodynamics,

$Q = \Delta U + W$ or $Q_1 - Q_2 = 0 + W$ or $W = Q_1 - Q_2$

Now $\eta = \frac{\text{Output}}{\text{Input}} = \frac{Q_1 - Q_2}{Q_1} = \frac{Q_1}{Q_1} - \frac{Q_2}{Q_1}$ or $\eta = 1 - \frac{Q_2}{Q_1}$

since $Q \propto T$, so $\eta = 1 + \frac{T_2}{T_1}$ & % age $\eta = \left(1 + \frac{T_2}{T_1}\right) 100$

The above equation shows that the efficiency of Carnot engine depends on the temperature of hot and cold reservoirs. It is independent of the nature of working substance. It can never be 100 %. All real heat engines are less efficient than Carnot engine.

Carnot's Theorem: *No heat engine can be more efficient than a Carnot engine operating between the same two temperatures.*

Extension of Carnot's Theorem: *All Carnot's engines operating between the same two temperatures have the same efficiency, irrespective of the nature of working substance.*

Thermodynamic Scale

The fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

Absolute zero is at -273.16° C, which is zero K. The degree intervals are identical to those measured on the Celsius scale. The unknown temperature T in Kelvin is, $T = 273.16\ Q/Q_3$
Thermodynamic scale is independent of the working substance. It is based on the principles of a reversible heat engine. Kelvin scale is based on the behaviour of an ideal gas.

Petrol engine: An engine based on the principle of Carnot cycle. It undergoes 4 processes;

i) Intake of petrol air mixture into the cylinder with a outward piston.
ii) Adiabatic compression of the mixture with the inward piston.
iii) A spark causing adiabatic expansion & piston delivers power to derive the flywheel.
iv) The residual gases are expelled from the outlet valves and piston moves inward.

Diesel engine: Its like a petrol engine but without sparkplug. It also undergoes four processes.

Entropy & Second Law of Theromodynamics

The physical quantity which describes the ability of a system to do work and it also describes disorder of a system. Mathematically $\Delta S = \Delta Q / T$

Entropy is a state variable. The more disordered the state of a system, the larger will be its entropy. Change in entropy is positive when heat is added and negative when heat is removed from the system.

Another form of Second Law of Thermodynamics

If a system undergoes a natural process, it will go in the direction that causes the entropy of the system plus the environment to increase.

Environmental Crisis

Global level problems created by man in the physical environment and by use of physical resources. It concerns with temperature, humidity, atmosphere and contamination.

Environmental crisis is an entropy or disorder crisis resulting from our futile efforts to ignore the second law of thermodynamics. It causes to disrupt ecological balance.